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SPECTROGRAPHIC OBSERVATIONS OF STANDARD VELOCITY STARS (1902–1903).

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DURING the past year one part of the program of work with the Bruce spectrograph of the Yerkes Observatory has been the observation of the list of stars adopted for co-operative observation by those engaged in line-of-sight work with the largest spectrographs now in use. The scheme[1] of co-operation that seemed to represent the views of those interested called for three spectrograms per year of each of the ten stars (all having spectra of the solar type), taken if possible at about the date of the star's opposition and thirty days before and after opposition. It was recommended that additional observations should be made of *a Boötis* and *a Persei* throughout the year. The choice of comparison spectrum and mode of reduction was to be left wholly to the different observers, variety being considered desirable. A style of publication was suggested, uniform as regards the minimum amount of data to be communicated.

We have endeavored to carry out the plan of co-operation as

[1] "Co-operation in Observing Radial Velocities of Selected Stars," ASTROPHYSICAL JOURNAL, **16**, 169–177, October 1902.

closely as possible since July 1902, but the weather conditions on the two nights weekly allotted to the spectrograph have not permitted us to obtain the plates as nearly on the schedule dates as we desired. In fact, we were able to obtain but a single plate of *β Leporis* during the past winter. The list of stars, with the approximate dates at which they were to be observed, if possible, is as follows:

α Arietis,	Oct. 1, Nov. 1, Dec. 1.
α Persei,	Oct. 23, Nov. 23, Dec. 23.
β Leporis,	Nov. 10, Dec. 10, Jan. 10.
β Geminorum,	Dec. 12, Jan. 12, Feb. 12.
α Crateris,	Feb. 14, March 14, April 14.
α Boötis,	March 13, April 13, May 13.
β Ophiuchi,	May 15, June 15, July 15.
γ Aquilae,	June 22, July 22, Aug. 22.
ε Pegasi,	July 24, Aug. 24, Sept. 24.
γ Piscium,	Aug. 13, Sept. 13, Oct. 13.

With the exception above noted, we have obtained three measurable plates of each of the above stars, and several extra plates of *α Boötis*, but unfortunately no extra plates of *α Persei*. We have also secured a few plates of the supplementary stars proposed by M. Bélopolsky as substitutes for the southern stars of the above list, which would be beyond his reach. Control plates of the Moon and planets have also been taken at intervals to test the performance of the spectrograph. The results for these plates are given below for the period covered by the plates of standard stars.

A detailed description of the Bruce spectrograph, with illustrations, has been given in this JOURNAL,[1] and need not be repeated here. The principal changes which have been made since that article was written have been: (1) the use of the lens instead of the mirror for projecting on the slit the image of the spark which furnishes the comparison spectrum; (2) the use of a small disk of ground glass at a distance of 20 mm in front of the slit, for diffusing the light of the spark, and insuring the complete and uniform illumination of the collimator by the rays from the spark; (3) a Hastings isokumatic collimator lens has

[1] 15, 1–27, January 1902.

been substituted for the triple of the same size (51 mm) and focal length (958 mm) at first used, resulting in some slight improvement in the previously excellent performance of the collimator.

The camera lenses which have been employed in the work included in this paper are designated as A, B, and B′. A is a Zeiss anastigmat of the "Protar" type, of 71 mm aperture and 449 mm focus, purchased of Bausch and Lomb; B is a triple lens, designed by Professor Hastings and constructed by Brashear, of 76 mm aperture and 607 mm focus; and B′, a triple from the same source as B, is of 57 mm aperture and of the same focus as B.

As the scale of the plates taken with camera B is one-third larger than for A plates, and as their definition is somewhat better at the center, we have naturally preferred to use camera B, which practically requires the same exposure time as A. B, however, has at times shown a disposition to become astigmatic, probably due to pressure on its cemented surfaces within its cell, and A has always been used when the trial plates suggested any lack of sharpness in the lines given by B. Hence A has been employed for about one-third of the plates concerned in this paper. Despite its smaller scale, camera A performs quite as well, if not better than B, as inferred from the accordance of the velocities deduced from different lines on the same plate. (See the values of ϵ in the details of measurements.) Camera B′ is also a cemented system, but has 25 per cent. less aperture than B, and is also mounted in a spring cell, so that no untoward effects are to be feared from pressure.

The performance of the great refractor has been most satisfactory in work with the spectrograph. The addition of a long-focus finder, without tube, having a six-inch object-glass attached beside the cell of the forty-inch lens with an eyepiece near the spectrograph, enables the observer to bring a star very quickly upon the slit, regardless of the flexure of the great tube.

The temperature-case of the spectrograph has also operated in a very satisfactory manner, as will appear from an examination of the records of the temperature in the outer and inner cases given in the journal of observations.

Additional details regarding the instrument, together with a new photograph of it and its accessories, may be found in our paper on the "Radial Velocities of Twenty Stars having Spectra of the *Orion* Type" in the forthcoming Vol. II of the *Publications of the Yerkes Observatory*.

The dispersion and scale of the plates for cameras A and B are as follows:

Wave-Length	Angular Dispersion for One Tenth-Meter	Tenth-Meters per mm	
		Camera A	Camera B
4300	45.7	10.0	7.4
4500	33.8	13.5	10.1
4700	26.0	17.6	13.1

On well-exposed spectra of the solar type a sufficient number of good stellar lines is usually found within a range of about 130 tenth-meters (or 13 mm for camera B), with the center at about λ4480, which is the ray passing the prisms at the angle of minimum deviation.

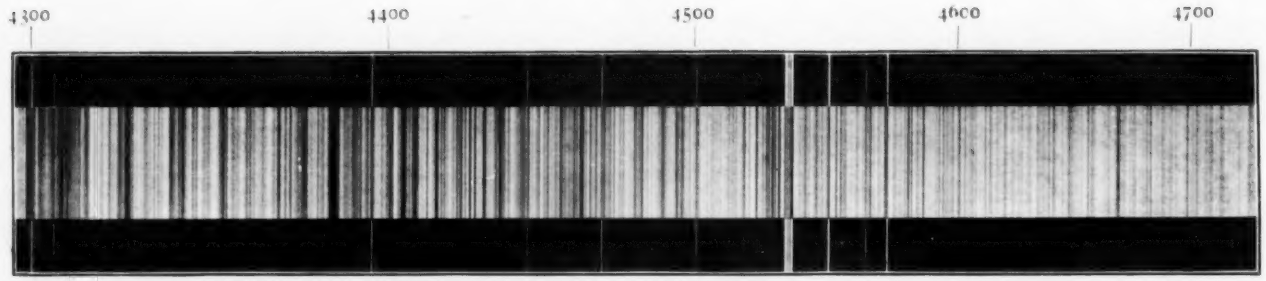
The titanium spark has been chiefly employed for the comparison spectrum on these plates, but electrodes of iron and chromium, and a helium tube can be readily rotated into the proper position for use when desired. Plate VIII, taken from the already cited article in the *Publications* of this Observatory, illustrates the distribution of the comparison lines of the *Ti* spectrum, and shows the quality of stellar spectrum obtainable. The use of a small self-induction coil obviates the presence of any air lines in the comparison spectrum, which, without the coil, are less pronounced with *Ti* than with most metallic electrodes.

MEASUREMENT OF THE PLATES.[1]

The plates have been measured with two screw-comparators constructed by Wm. Gaertner & Co., of Chicago. The screws have a pitch of 0.5 mm, and the head is large, so that the single

[1] An extended discussion of this topic, and of the various sources of errors in our determinations of radial velocities, is given in the volume of the *Publications* already alluded to, which has at this writing been in type for about eight months. For details the reader is referred to that paper.

PLATE VIII.



α Boötis, with Titanium Comparison Spectrum.

Enlarged vertically fortyfold, horizontally fourfold.

divisions (0.001 mm) can be easily read through a small magnifier and estimated to tenths. A distance of less than 20 mm is ordinarily measured on the spectra here in question—in fact, for A plates usually only about 10 mm, and for B plates about 13 mm. Four settings are made on each of the comparison and stellar lines which the observer desires to employ, and then the plate is reversed under the microscope (so that the violet side shall be toward the right instead of the left), and the process is repeated.

The measurements in the two positions are combined to means, and then reduced with the aid of the simple Hartmann formula, the constants of which are determined from three standard comparison lines selected near the middle and ends of the portion of spectrum measured. The computed wave-lengths of the various other comparison lines distributed along the plate, always selected so as to be as near as possible to the best star lines, serve to give successive checks upon the accuracy of the "fit" of the formula. The corrections to formula thus obtained are applied to the corresponding or intervening star lines, with such (linear) interpolation as may be necessary. We have feared the dangers of abitrariness in any process of smoothing out the accidental errors of settings by a curve, so that the accordance of the velocities deduced from different lines on the same plate is less than it might be otherwise; but the mean of the values probably may be regarded as quite as trustworthy, and is certainly free from errors due to prepossession in curve-drawing. From the values of the apparent wave-lengths of the stellar lines thus derived, the displacements, and hence the corresponding velocities, are immediately inferred. It is hardly necessary to remind the reader that the full amount of any error in the relative wave-length[1] of any line enters into our result, a relative error of 0.01 tenth-meter producing an error of 0.7 km per second in the velocity deduced for that line.[2] It will, however, be easy to correct our results later, as subsequent laboratory

[1] Taken from ROWLAND'S "Preliminary Table of Solar Spectrum Wave-Lengths."

[2] Unless the star line is identical with the comparison line, so that a direct displacement is measured.

observations shall gradually remove the relative errors in Rowland's tables. In our computations we carry the wave-lengths to the third decimal place, as given by Rowland, but of course we do not attach significance to measures on any given plate beyond the hundredth of a tenth-meter. Similarly, though we carry out our reductions to the hundredth of a kilometer, it is clear that the uncertainties of the wave-lengths, quite aside from the accidental errors of measurement, seriously affect the tenths of a kilometer.

Tests made by ourselves and by others indicate that the measurement of the plate in the two positions under the microscope largely eliminates the physiological error due to the difference in the nature of the process of setting a dark thread upon a white star line and the same dark thread upon a dark comparison line (on the negative). Our systematic errors of this sort would be quite large, about 4 km for Frost and 2 km for Adams, but with opposite signs.

Preliminary investigations of the periodic errors of the screws of the two comparators were made during the progress of this work, and errors were found, reaching a maximum of 4μ for one machine and of 3μ for the other. It was found that they changed when the nut was more tightly clamped to the screw, which was occasionally necessary, as the screws tapered slightly from both ends toward the center. Corrections have therefore not been applied for the errors of the screws in this year's work, and new screws are being supplied to replace the present ones. However, for reasons fully set forth in our article on the *Orion* stars, it does not appear to us probable that the radial velocity of any solar star deduced from measures of one plate will be affected by over 0.1 km on account of screw-errors. The general accordance in the measures of the same plate by the two observers with the two different machines is confirmatory of this opinion.

We desire to emphasize the independent character of the measurements of a plate by the two observers. The selection of the lines of both comparison and stellar spectrum was made by the measurer without reference to his own or the other observer's

measures of other plates of the same star, and no comparisons were made until all the reductions were completed. There has been no attempt to secure conformity in the habit of the two observers in making settings. Thus, for instance, A. is accustomed to make his settings on the inner tips of the comparison lines, while F. has followed the practice of setting upon a certain point estimated as one-third of the length of the comparison line from the inner tip. Of course, the corrections for curvature[1] have been different for these two distances from the center of the star spectrum, which distance is the argument in our table of curvature corrections. But if any serious error had entered in the habit of either observer in regard to the point at which the setting is made, it would appear as a persistent systematic difference in the results, which we cannot discover.

It is not to be doubted that the observers' mode of setting may change with time and experience, but we have not found any such changes, of magnitude sufficient to be regarded as real, during our work since the Bruce spectrograph has been in operation. The differences that occur in the results of the independent measurements of the same plate by F. and by A., tabulated below, are perhaps disappointingly large; and the deviations of the separate plates from the mean may be larger than would be anticipated by those who have not directly engaged in line-of-sight work; but they seem in any event to be of an accidental character. It is perhaps safe to say that at present, with a stable spectrograph, errors due to personality are quite as much to be feared as errors due to instrumental sources.

We have elsewhere[2] discussed the effect of our assumption of solar wave-lengths for spark lines. If arc and spark wave-length are absolutely identical, we might expect that a systematic correction to our results would be required to compensate for the displacement of lines due to pressure in the stellar atmosphere. There are not at present sufficient observational data available for making such corrections (to the solar wave-lengths),

[1] These are applied to the mean of the settings on each comparison line, and do not further appear in the reductions.

[2] *Loc. cit.*, p. 155.

and furthermore our control plates on the Moon and the planets do not reveal such a systematic difference from the computed velocities.

The excellent character of many lines in the stellar spectra, which are made up of two or more lines too close to one another to be separated with the dispersion employed, has necessitated the use of a number of blends. In all such cases we have followed the usual practice of assigning weights to the component lines according to the intensities given them in Rowland's table, and taking the weighted mean of their wave-lengths as the wave-length of the compound line. The maximum distance between two lines for which blending is admissible depends upon the practical separating power of the instrument. On lantern-slide plates of the solar spectrum the least distance at which two lines can be seen as distinct is about 0.15 tenth-meter. The less fine-grained plates used for stellar spectra do not admit of the resolution of such close lines, and with them the limit is usually from 0.25 to 0.30 tenth-meter, according to the quality of the plate and the character of the lines involved. In addition to the matter of separation, it is, of course, necessary to determine the intensity which a line must have in order to exert an appreciable influence upon a closely adjacent line. The answer to such a question is naturally somewhat difficult; an examination, however, of solar and planetary plates has led us to the conclusion that, upon a plate of quality suitable for good measurement, a line of intensity 0 on Rowland's scale is appreciable, while fainter lines may, in general, be neglected. A few exceptions are found to this rule, but these occur mainly in cases of uncommonly strong lines, when the influence of the faint component appears to be negligible.

The arbitrary character of the assumption of equal intensities for the same lines in the Sun, and in stars which are classed broadly under the solar type, finds its justification in the agreement of the values deduced from lines whose wave-lengths have been obtained under this assumption with those derived from uncompounded lines whose wave-lengths are subject to no uncertainty due to this cause; and also in the accord of the com-

pounded lines with the same lines in planetary and lunar spectra where the above method of blending is evidently rigorously correct. In the case of the stars discussed in this paper the wave-lengths of the blended lines, found in the way described above, have proved quite satisfactory for all except *a Persei*. The spectrum of this star, however, is of a less developed type than that of the other stars, its most striking characteristic being the prominence of the "enhanced" metallic lines. When such lines, accordingly, have formed the most important part in the composition of the blends used for the other stars, the wave-length of the enhanced line alone has in each case been used, and the results obtained have shown this to be unquestionably the correct procedure.

The table which follows contains the wave-lengths of all of the blended lines which we have employed two or more times in the course of our reductions, together with the elements to which they are due. Those which have been employed but once are indicated, where they occur in the tables of detailed reductions, by the letter B, following the wave-length of the line.

Elements	Wave-Length	Elements	Wave-Length	Elements	Wave-Length
Ni; Ti, Cr	4399.903	*Ni; Fe*	4466.701	*—; Cr; Fe*	4526.644
V; Fe; V	4408.549	*Fe; Ag*	76.214	*Ti;—*	27.518
Ti; Fe	27.420	*—, Fe; Fe*	82.376	*Ti–Co;—*	34.168
Fe; Ti	34.021	*Ni; Fe,—*	90.900	*Ni; Fe; Ni*	47.196
Ca; Fe	35.184	*Cr; Zr*	97.046	*Ni, Ti; Fe*	60.225
Mn; Ca	56.030	*Cr, Mn; Ti*	4501.422	*Cr; Co–Fe*	65.750
Ti; V, Zr; Mn	57.656	*—; Fe*	15.475	*Ca; Co, Fe*	81.634
Ni; Fe; Fe, Cr	59.304	*Fe?;—Ti*	22.853	*Cr; Fe*	4611.455
V; *Mn;—*	60.460				

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The table which follows contains the observational data for all of the plates discussed in this article. As usual the series letters A, B, and B′ refer to the three cameras already described earlier in this paper. The middle of the star exposure is given in Greenwich Mean Time, and the hour angle at the end of the exposure is added to indicate the position of the telescope at the time.

The exposure of the plate to the comparison spectrum has commonly been made at the beginning and end of the star

Object	Series and Number	Date	Middle of Exposure	Duration	Hour Angle at End	Slit-Width
		1902	h m	m	h m	mm
Moon	A 350	July 16	14 28	40	E 0 20	0.030
ε Pegasi	A 364	July 31	19 15	62	W 0 45	0.032
β Ophiuchi	B 378	Aug. 7	15 06	77	W 1 15	0.038
ε Pegasi	B 379	Aug. 7	16 37	70	E 1 20	0.038
γ Piscium	B 381	Aug. 7	19 23	120	W 0 20	0.038
α Persei	B 382	Aug. 7	20 53	30	E 3 00	0.020
γ Aquilae	B 398	Aug. 27	17 02	100	W 2 40	0.038
Moon	B 401	Aug. 27	15 47	37	E 3 25	0.038
α Boötis	A 373	Sept. 6	7 43	15±	W 4 50	0.025
Moon	B 408	Sept. 13	10 13	35	W 1 32	0.038
γ Piscium	B 415	Oct. 8	14 38	153	E 0 05	0.038
γ Aquilae	B 417	Oct. 9	14 12	125	W 2 55	0.038
ε Pegasi	B 418	Oct. 9	16 14	93	W 2 40	0.038
α Arietis	B 420	Oct. 9	19 08	40	W 0 45	0.038
Moon	B 423	Oct. 15	15 44	30	E 0 46	0.028
γ Cephei	B 428	Oct. 16	17 44	70	W 2 25	0.038
α Arietis	B 430	Oct. 29	13 23	50	E 3 35	0.038
α Persei	B 431	Oct. 29	14 21	40	E 4 00	0.038
γ Piscium	B 436	Oct. 30	14 39	120	W 1 05	0.038
Moon	B 444	Nov. 6	13 38	40	W 3 15	0.032
ι Aurigae	B 446	Nov. 6	15 31	75	E 3 35	0.032
β Leporis	B 449	Nov. 6	19 55	90	W 0 25	0.038
α Persei	B 458	Nov. 19	13 27	33	E 3 35	0.038
γ Cephei	B 464	Nov. 27	10 35	75	W 4 45	0.038
α Arietis	B 465	Nov. 27	18 35	52	W 3 30	0.038
β Geminorum	B 477	Dec. 31	20 13	20	W 1 30	0.038
Mars	B 478	Dec. 31	20 56	32	E 2 25	0.038
		1903				
ε Leonis	B 483	Jan. 8	21 56	88	W 2 20	0.038
Moon	A 387	Jan. 16	14 28	25	E 0 30	0.038
α Crateris	B 491	Feb. 4	19 55	148	W 1 23	0.045
α Boötis	B 492	Feb. 4	21 39	20	E 1 25	0.045
β Geminorum	A 398	Feb. 5	19 31	18	W 3 05	0.038
Mars	A 400	Feb. 5	22 15	50	W 0 50	0.038
α Boötis	B 497	March 24	22 48	13	W 2 55	0.038
γ Cephei	A 424	April 3	15 41	68	W 11 28	0.038
β Geminorum	A 426	April 8	15 14	27	W 3 00	0.038
ε Leonis	A 427	April 8	16 11	70	W 2 20	0.038
Moon	A 429	April 8	18 35	25	W 3 05	0.038
α Boötis	A 433	April 8	21 46	16	W 2 50	0.030
α Crateris	A 438	April 16	17 24	132	W 3 18	0.038
ε Leonis	A 443	April 22	14 48	52	W 1 40	0.030
α Crateris	A 444	April 22	16 56	157	W 3 25	0.042
Mars	A 445	April 22	18 46	35	W 3 10	0.030
β Ophiuchi	A 451	April 30	19 55	95	E 0 20	0.038
α Boötis	B′ 499	May 6	14 44	20	E 2 15	0.038
γ Cephei	B′ 501	May 6	17 42	100	E 8 00	0.038
Mars	B′ 502	May 6	19 20	50	W 5 00	0.038
Venus	B′ 505	June 16	14 11	9	W 5 10	0.020
β Ophiuchi	B′ 508	June 26	15 32	83	E 1 00	0.040
γ Aquilae	B′ 509	June 26	17 25	110	E 0 55	0.040

Series and Number	Comparison Beginning	Comparison End	Temperature Beginning *i*	Temperature Beginning *o*	Temperature End *i*	Temperature End *o*	Seeing	Observer	Remarks
A 350	*Ti* 5	*Ti* 5	23.4	23.9	23.6	24.2	2; 1	A	Light cirrus
A 364	*Ti* 45	*Ti* 45	25.5	25.4	25.5	25.4	3; 3	A	clouds.
B 378	*Ti* 30	*Ti* 30	21.5	21.5	21.5	21.5	3–4; 3–4	F	
B 379	*Ti* 20	*Ti* 33	21.5	21.4	21.4	21.1	4; 3–4	F	
B 381	*Ti* 24	*Ti* 28	21.5	21.5	21.5	21.5	4; 4–3	F	
B 382	*Ti* 80	*Ti* 19			21.4	21.1	4; 4	F	
B 398	*Ti* 50, *Cr* 90	*Ti* 40, *Cr* 90	22.2	22.2	22.2	22.2	3; 3–4	A	Image very
B 401	*Ti* 40	*Ti* 45	22.2	22.4	....	....	3; 3	A	unsteady.
A 373		*Ti* 115	20.3	20.2	....	....	3–0; 2	A	Frequent
B 408	*Ti* 40	*Ti* 40	12.7	12.7	12.7	12.4	3; 1	A	clouds.
B 415	*Ti* 45	*Ti* 35	20.1	20.0	20.1	20.3	3; 2	A	
B 417	*Ti* 20 at 13:35	*Ti* 40	12.4	12.2	12.4	12.3	2; 2	F	
B 418	*Ti* 30	*Ti* 15	12.4	11.8	12.4	12.0	2; 2–3	F	
B 420	*Ti* 20	*Ti* 40	12.4	12.0	12.3	11.8	2; 2	F	
B 423	*Ti* 30	*Ti* 40	15.4	15.4	15.4	15.3	3; ...	A	
B 428	*Ti* 20	*Ti* 20	10.3	10.0	10.2	9.9	2–3; 2–3	F	
B 430	*Ti* 40	*Ti* 35	7.4	7.6	7.5	7.6	2; 2–1	A	
B 431	*Ti* 40	*Ti* 25	7.5	7.6	....	....	2; 2	A	
B 436	*Ti* 30	*Ti* 30	12.8	12.4	12.8	12.5	4; 4	F	
B 444	*Ti* 60		7.1	7.1	7.1	6.9	3; ...	F	
B 446	*Ti* 30	*Ti* 30	7.2	7.1	7.3	7.1	4; 4	F	
B 449	*Ti* 25	*Ti* 25	7.2	7.3	7.2	7.3	4; 4	A	
B 458	*Ti* 35	*Ti* 25	9.8	9.9	9.8	9.8	3; 3–4	A	Heavy dew on
B 464	*Ti* 25	*Ti* 30	− 0.2	0.0	− 0.1	+ 0.4	4; 3	F A	object-glass.
B 465	*Ti* 25		− 0.1	− 0.6	− 0.1	− 0.4	4; 4–3	F	Star's light
B 477	*Ti* 20	*Ti* 18	− 1.2	− 1.2	− 1.2	− 1.3	4; 2	A	cut down by
B 478	*Ti* 24	*Ti* 16	− 1.3	− 1.4	− 1.3	− 1.2	4; 1	A	snow clouds
									after 15:10.
B 483	*Ti* 25	*Ti* 25	−10.2	−10.4	−10.2	−10.3	3; 3	A	
A 387	*Ti* 20	*Ti* 20	− 0.3	− 0.1	− 0.4	− 0.8	3; ...	A	
B 491	*Ti* 15	*Ti* 15	− 4.8	− 4.8	− 4.8	....	3; 2	F	
B 492	*Ti* 15	*Ti* 15	− 5.0	....	− 4.9	− 4.6	3: 2	F	
A 398	*Ti* 25	*Ti* 25	− 6.1	− 6.0	− 6.1	− 6.3	3; 4	A	
A 400	*Ti* 23	*Ti* 20	− 6.1	− 6.4	− 6.1	− 6.6	3; 3	A	
B 497	*Ti* 40		0.0	0.0	....	....	2; 2	A	
A 424	*Ti* 30	*Ti* 30	1.3	1.2	1.4	1.4	3; 3	F	
A 426	*Ti* 30	*Ti* 25	14.7	14.8	14.8	14.8	3; 1	A	
A 427	*Ti* 25	*Ti* 25	14.8	14.8	14.8	14.8	3; 1–2	A	
A 429	*Ti* 25	*Ti* 25	14.8	14.9	14.8	14.5	3; ...	A	
A 433	*Ti* 32	*Ti* 30	14.8	14.9	14.8	14.8	3; 2	A	
A 438	*Ti* 20	*Ti* 25	8.3	8.0	8.2	7.8	4; 3	A	
A 443	*Ti* 30	*Ti* 35	9.7	10.5	10.2	9.7	4; 4+	F	
A 444	*Ti* 25	*Ti* 30	9.9	10.4	9.9	10.1	3; 4	F	
A 445	*Ti* 25	*Ti* 35	9.9	10.0	9.9	9.3	3; 4	F	
A 451	*Ti* 25	*Ti* 25	4.5	4.4	4.5	4.0	3–0; 3	A	Cloudy from
B′ 499		*Ti* 120	13.1	12.6	13.1	12.9	3; 3	F	13:00–
B′ 501	*Cr* 45, *Ti* 40	*Ti* 55, *Cr* 40	13.1	13.0	13.1	12.3	2–3; 2–3	F	13:35.
B′ 502	*Cr* 60, *Ti* 80		....	....	13.2	12.4	2–1; 2	F	
B′ 505	*Ti* 180, *Cr* 150		19.5	19.6	....	....	3; 2	A	
B′ 508	*Ti* 50	*Ti* 85	23.3	22.9	23.1	22.5	2; 3	F	
B′ 509	*Ti* 45	*Ti* 65	23.1	24.5	23.1	23.3	3; 3–4	F	

exposure, and the length in seconds is given by the number following the symbol of the element employed. The temperature conditions during the star exposure are indicated in the column headed "Temperature." The letter *i* refers to the thermometer within the prism box, *o* to the thermometer in the large outside temperature case. The readings are given only for the beginning and end of the exposure, as these indicate in the case of all the plates considered here the maximum range of temperature through which the air within the prism-box has passed. The last column of the table gives the estimated transparency of the sky and the steadiness and quality of the star's image, in the order named, and on a scale of 0 to 5, 5 denoting perfect conditions in each case.

The kind of photographic plate employed is not stated in the journal of observations. Previous to December 1902, Seed's "Gilt Edge 27," were used almost exclusively. Since that time the majority of photographs have been made with Seed's double-coated non-halation plates, with the outer emulsion also of the "Gilt Edge 27" quality. The principal developers used have been rodinol and edinol.

We are indebted to Mr. F. R. Sullivan, engineer in charge of the telescope, for much efficient assistance in the labor of guiding during the exposures entered in the list.

DETAILED REDUCTIONS.

The tables of detailed reductions which follow are divided into two parts, the first containing the results given by the plates of the Moon and the planets which have been taken as a check upon the behavior of the spectrograph, and the second the results for the stars whose discussion forms the subject-matter of this article. We have endeavored, so far as possible, to make the form of tabulation the same for both.

The time and hour angle given for each plate refer to the middle of the exposure. The former is expressed in Greenwich Mean Time. The first column of the table gives the wave-length employed for the stellar line, and following it in parallel columns is the velocity found for that line by each observer. The

mean for each observer is given to hundredths of a kilometer in order to avoid error in the formation of the final mean, but the latter is given only to tenths of a kilometer. On account of the great amount of repetition involved, it has not seemed desirable to give the wave-lengths of all the comparison lines used in the reduction of the plates, but the number of lines so used, as well as the wave-lengths of the standard lines employed in the determination of the constants of the reduction formula, is found at the foot of the table for each plate.

All of the velocities given, as well as the mean errors, are, of course, expressed in kilometers per second.

CONTROL PLATES OF THE MOON AND THE PLANETS.

The series of Moon and planet plates given below covers practically the whole interval over which the stellar results given in this paper extend. The first five plates of the Moon are summarized in a short table, the details of their reduction being given in Vol. II of the *Publications of the Yerkes Observatory*.

The theoretical velocities both of the Moon and of the planets have been computed with the aid of the formulæ given in so convenient a form by Professor Campbell.

One plate of *Mars*, B′502, has been excluded from the list given, as it was found on development that the glass was broken diagonally across, the photographic film alone holding the fragments together.

MOON.

Series and Number	Date	Hour Angle	Measured by	Number of Lines	Mean	Computed Velocity	O.–C.	Quality of Plate
A 350	1902, July 16, 14^h 28^m	E 0^h 42^m	A.	13	+0.4	+0.4	0.0	Fair
B 401	Aug. 27, 21 47	E 3 44	A.	14	−1.1	−1.5	+0.4	Fair
B 408	Sept. 13, 16 13	W 1 14	A.	12	+0.1	+0.2	−0.1	Good
B 423	Oct. 15, 15 44	E 1 01	A.	13	−0.6	−0.4	−0.2	Good
B 444	Nov. 6, 13 38	W 2 54	A.	14	+0.7	+0.6	+0.1	Good

MARS—B 478.

1902, December 31, 20^h 56^m. Hour angle, E 2^h 41^m. Measured by A.
Planet spectrum a trifle weak; comparison spectrum weak.

Line, Wave-Length in Sun	Velocity	Line, Wave-Length in Sun	Velocity
4468.663	−17.11	4565.750	−17.07
82.376	14.94	81.634	17.27
4501.422	17.92	84.018	17.66
28.798	16.49	4611.455	19.44
48.938	18.06		
54.211	15.74		
63.939	18.33		(11)
Number of comparison lines ..	9		
Ti standards used	4468.663 4534.139 4617.452	Mean Computed velocity	−17.3 −17.1

MOON—A 387.

1903, January 16, 20^h 40^m. Hour angle, E 0^h 42^m.
Moon spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	+0.14	−1.56	4494.738	−3.60	−1.27
42.510	−1.15	−1.82	97.046	−3.60	−1.53
43.976		−1.62	4501.422	−2.13	−1.33
47.892	+0.94	0.00	22.853		+0.27
56.030	−1.95		28.798	+0.33	−0.73
57.656	−2.49	+0.87	34.168		−1.19
68.663	−0.40		54.211	−1.51	
76.214	−0.07				
82.376		−0.80		(12)	(12)
No. of compar. lines	11	11	Means	−1.29	−0.89
Ti standards used	4427.266 4481.438 4555.662	4427.266 4481.438 4527.490	Mean Computed velocity	−1.1 −0.6	

MARS—A 400.

1903, February 15, 22^h 15^m. Hour angle, W 0^h 25^m.

Planet spectrum fair; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	−15.85	−16.46	4494.738	−16.21	−15.61
42.510	14.31	14.65	4501.422	14.32	16.12
43.976	12.69	13.97	28.798	14.37	15.03
47.892	13.48	14.56	34.168		15.21
56.030	14.47		54.211	19.62	
57.656	15.20	15.40			
68.663	13.76	12.55			
76.214	10.38				
82.376		13.98			
90.900		16.09		(12)	(12)
No. of compar. lines	13	9	Means	−14.56	−14.97
Ti standards used	4427.266 4481.438 4555.662	4427.266 4481.438 4527.490	Mean −14.8 Computed velocity −15.5		

MOON—A 429.

1903, April 8, 18^h 35^m. Hour angle W 2^h 53^m. Measured by A.

Moon spectrum fair; comparison spectrum good.

Line, Wave-Length in Sun	Velocity	Line, Wave-Length in Sun	Velocity
4427.420	+2.91	4482.376	+2.74
42.510	2.16	97.046	2.00
47.892	2.97	4501.422	0.60
56.030	1.55	17.702	3.19
57.656	2.35	28.798	0.20
68.663	0.00		
76.214	0.54	.	(12)
Number of comparison lines...	10		
Ti standards used.........	4427.266 4481.438 4527.490	Mean +1.8 Computed velocity +1.3	

MARS—A 445.

1903, April 22, $18^h 46^m$. Hour angle W $2^h 52^m$.

Planet spectrum fair; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+4.13	+3.39	4476.214	+7.70	+6.10
35.184	5.81		82.376	7.29	6.49
41.881		6.21	94.738	3.94	
42.510	5.87	5.87	97.046	1.53	3.20
43.976		7.02	4501.422		3.73
47.892	5.19	4.18	15.475	3.92	6.57
56.030	5.18	4.78	26.644		6.82
57.656		5.45	28.798	6.42	4.37
59.304	5.31		54.211	7.04	
68.663	2.64	3.22		(14)	(15)
No. of compar. lines	14	11	Means	+5.14	+5.16
Ti standards used	4427.266	4427.266			
	4496.318	4481.438	Mean	+5.2	
	4555.662	4527.490	Computed velocity	+5.2	

VENUS—B 505.

1903, June 16, $14^h 11^m$. Hour angle W $5^h 6^m$. Measured by A.

Planet spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity	Line, Wave-Length in Sun	Velocity
4427.420	−13.39	4482.376	−11.44
42.510	11.53	94.738	14.01
43.976	11.32	97.046	13.07
47.892	11.58	4501.422	11.99
56.030	12.57	26.644	13.99
57.656	13.31	28.798	14.05
68.663	14.55		
76.214	11.72		(14)
Number of comparison lines...	10		
Ti standards used.........	4427.266	Mean	−12.8
	4481.438	Computed velocity	−13.2
	4527.490		

The mean value of O.−C. for the above eleven plates taken with regard to sign is +0.10 km. A quantity of this size might easily be due to accidental causes, so that the conclusion may

be drawn that the control plates show no evidence of the existence of any systematic error in the spectrograph.

STANDARD VELOCITY STARS.

The following tables give the results of our measures upon the plates of standard velocity stars. The ten stars of the principal list are arranged in order of right ascension, followed by three stars which we have observed in the supplementary list.

In addition to the data given for the control plates, we append, for the star plates, the mean error ϵ of the determination of the velocity from a single line for each observer, and the mean error ϵ_0 of the final velocity of the star deduced by each observer from one plate, viz.:

$$\epsilon = \pm\sqrt{\frac{\Sigma v^2}{n-1}}\,, \quad \text{and} \quad \epsilon_0 = \pm\sqrt{\frac{\Sigma v^2}{n(n-1)}}\,.$$

The reduction of the star's velocity to the Sun is given in its two parts, V_a denoting the correction due to the Earth's orbital motion, and V_d the correction due to the Earth's diurnal rotation. Schlesinger's tables of star constants for the determination of V_a have been employed throughout.

Reference should be made to the fact that all of the measurable plates obtained are included in the reductions given below, and that none have been omitted because of inferior quality or lack of agreement with other plates.

α *ARIETIS* — B 420.

1902, October 9, $19^h\ 08^m$. Hour angle W 0^h 25

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−23.03	−23.37	4482.376	−21.34	−22.82
28.711		21.53	94.738	23.68	24.02
41.881		23.70	97.046	22.40	23.74
42.510	23.42	22.89	4501.422	24.25	23.58
43.976		23.62	12.906	21.33	
47.892	22.38		26.644		23.12
56.030	24.09	22.01	28.798	22.12	23.64
60.460		23.20	48.938	23.40	
68.663	21.20		54.211	24.29	
76.214	23.65	21.78		(14)	(15)
No. of compar. lines	12	11	Means	−22.90	−23.07
ϵ	±1.10	±0.78	Mean	−22.98	
ϵ_0	±0.29	±0.21	V_a	+9.70	
Ti standards used	4427.266	4427.266	V_d	−0.03	
	4481.438	4481.438	Reduction to Sun	+ 9.67	
	4527.490	4527.490	Radial velocity	−13.3	

α *ARIETIS* — B 430.

1902, October 29, $13^h\ 23^m$. Hour angle E $4^h\ 0^m$.

Star spectrum very good; comparison spectrum very good.

4427.420	−13.82	−13.75	4499.900	−16.49	
28.711		15.23	94.738	15.41	−16.28
42.510	12.02	13.64	97.046	11.14	11.80
43.976		13.77	4501.422		15.59
47.892	12.81	13.14	12.906	16.15	16.01
57.656	12.65	11.70	15.475	14.67	
60.460	14.05	16.13	17.702		15.46
68.663	10.06	12.41	26.644	13.38	
76.214	14.40	16.48	28.798	11.98	12.12
82.376	13.65	12.38			
				(15)	(16)
No. of compar. lines	12	11	Means	−13.51	−14.12
ϵ	±1.80	±1.74	Mean	−13.82	
ϵ_0	±0.46	±0.44	V_a	−0.14	
Ti standards used	4427.266	4427.266	V_d	+0.27	
	4481.438	4481.438	Reduction to Sun	+ 0.13	
	4527.490	4527.490	Radial velocity	−13.7	

α *ARIETIS* — B 465.

1902, November 27, $18^h 35^m$. Hour angle W $3^h 4^m$.

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+0.07	+1.08	4482.376	+2.27	+2.21
28.711		−0.20	94.738	−2.13	−0.87
41.881		−0.47	97.046	+0.73	+1.00
42.510	+1.69	+2.23	4501.422	−0.80	+0.60
43.976		−0.74	12.906	+2.53	+1.26
47.892	−0.13	0.00	28.798	+2.05	+0.86
56.030	+1.35	+0.54	54.211	+0.07	
57.656	+0.81	+2.35			
60.460		−0.87			
68.663	+1.68	+2.42			
76.214	+0.94	+0.40		(14)	(17)

	F.	A.			
No. of compar. lines	13	11	Means	+0.80	+0.69
ϵ	±1.29	±1.14	Mean	+ 0.74	
ϵ_0	±0.34	±0.28	V_a	−14.58	
Ti standards used	4427.266	4427.266	V_d	− 0.23	
	4481.438	4481.438	Reduction to Sun	−14.81	
	4555.662	4527.490	Radial velocity	−14.1	

α *PERSEI* — B 382.

1902, August 7, $20^h 53^m$. Hour angle E $3^h 15^m$.

Star spectrum fair but unsymmetrical; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4443.976		− 24.36	4515.508	−25.16	−25.76
47.892		27.71	20.397	26.54	
59.301	−26.02		22.802	26.19	24.40
66.727	25.04	26.52	28.798	27.41	28.01
68.663	25.50	24.56	54.211	26.53	25.61
76.214	27.47	28.07	58.827		23.74
89.351	28.99		63.939		23.65
91.570	24.97		72.156		25.18
94.738	31.02	28.49			
4501.445	26.24	27.18			
08.455	27.93	28.20		(14)	(15)

	F.	A.			
No. of compar. lines	12	15	Means	−26.79	−26.10
ϵ	±1.68	±1.74	Mean	−26.44	
ϵ_0	±0.44	±0.45	V_a	+24.48	
Ti standards used	4455.485	4443.976	V_d	+ 0.17	
	4512.906	4501.445	Reduction to Sun	+24.65	
	4555.662	4563.939	Radial velocity	− 1.8	

α PERSEI — B 431.

1902, October 29, $14^h\ 21^m$. Hour angle E $4^h\ 20^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4443.976		−11.27	4508.455	−12.17	−11.24
44.728		12.68	15.508	11.69	10.82
47.892		10.85	20.397	12.88	13.80
59.301	−14.32		22.802	11.74	
66.727	14.23	15.30	28.798	13.18	13.91
68.663	11.94	11.07	34.139		10.71
76.214	10.45	12.06	39.759		13.74
94.738	12.61	13.61	54.211	11.39	13.10
4501.445	12.59	13.19		(12)	(15)
No. of compar. lines	12	12	Means	−12.43	−12.49
ϵ	±1.13	±1.44	Mean	−12.46	
ϵ_0	±0.32	±0.37	V_a	+10.60	
Ti standards used	4457.600	4449.313	V_d	+ 0.20	
	4512.906	4501.445	Reduction to Sun	+10.80	
	4555.662	4555.662	Radial velocity	− 1.7	

α PERSEI — B 458.

1902, November 19, $13^h\ 27^m$. Hour angle E $3^h\ 51^m$.

Star spectrum good; comparison spectrum slightly strong.

Line, Wave-Length in Sun	F.	A.	Line, Wave-Length in Sun	F.	A.
4427.420		−2.51	4515.508	−4.25	−2.92
43.976		2.36	20.397	6.63	6.24
44.728		6.61	22.802	3.45	
47.892		7.15	28.798	7.02	6.69
59.301	−2.02		34.139		3.37
66.727		4.23	41.690		2.51
68.663	4.50	4.23	54.211	6.52	2.70
76.214	7.17		58.827	4.41	5.99
89.351	3.81		63.939	4.73	3.68
91.570	2.80		72.156	4.98	
94.738		4.20			
4501.445	5.46	3.40			
08.455	4.99	6.98		(15)	(17)
No. of compar. lines	11	13	Means	−4.85	−4.46
ϵ	±1.47	±1.75	Mean	−4.66	
ϵ_0	±0.37	±0.42	V_a	+1.48	
Ti standards used	4457.600	4427.266	V_d	+0.19	
	4512.906	4481.438	Reduction to Sun	+1.67	
	4563.939	4555.662	Radial velocity	−3.0	

β LEPORIS—B 449.

1902, November 6, $19^h\ 55^m$. Hour angle E $0^h\ 20^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−24.04	−26.75	4501.422	−24.58	−22.98
42.510		23.56	08.455	25.41	25.21
43.976		23.55	15.475	25.30	26.83
47.892	22.92	22.65	25.285 B	26.50	
56.030		26.18	26.644		26.23
57.656	25.70	24.55	28.798	22.84	23.64
68.663	23.89	23.82	46.129	23.74	
76.214	24.19	23.45	54.211	24.56	
82.376	23.48	23.75			
94.738		26.49			
97.046	21.40	22.87		(14)	(16)
No. of compar. lines	11	11	Means	−24.18	−24.53
ϵ	±1.32	±1.50	Mean	−24.36	
ϵ_0	±0.35	±0.37	V_a	+11.95	
Ti standards used	4427.266	4427.266	V_d	+ 0.03	
	4481.438	4481.438	Reduction to Sun	+11.98	
	4555.662	4527.490	Radial velocity	−12.4	

β GEMINORUM—B 477.

1902, December 31, $20^h\ 13^m$. Hour angle W $1^h\ 20^m$.

Star spectrum good; comparison spectrum weak.

4427.420	−3.12		4522.853	−1.86	
42.510	3.44	−2.02	28.798	2.45	−2.32
43.976		1.15	34.168	2.18	1.92
47.892		1.75	47.196	2.51	
56.030		2.49	48.938	5.08	
57.656	2.09	1.68	54.211	3.23	2.30
60.460		1.01	63.939	3.28	3.55
68.663	2.89	2.48	71.275	5.90	2.36
76.214		2.55			
82.376		3.14			
97.046	3.87	4.33			
4501.422	5.33	3.86		(14)	(16)
No. of compar. lines	12	10	Means	−3.37	−2.43
ϵ	±1.28	±0.92	Mean	−2.90	
ϵ_0	±0.34	±0.23	V_a	+6.30	
Ti standards used	4427.266	4443.976	V_d	−0.10	
	4501.445	4501.445	Reduction to Sun	+6.20	
	4563.939	4563.939	Radial velocity	+3.3	

β GEMINORUM—A 398.

1903, February 5, 19h 31m. Hour angle W 2h 56m

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4399.903		+13.83	4460.460		+15.60
4406.810		16.47	68.663	+14.29	16.64
07.851 B		17.14	76.214	14.27	
08.549		16.39	82.376	15.45	16.66
11.240		14.55	94.738		15.68
27.420	+15.58	17.95	97.046	15.60	14.94
42.510	16.60	16.33	4501.422	14.45	15.39
43.976		14.85	12.906	15.08	17.61
47.892	16.45	14.83	22.853	17.30	
56.030		17.83	28.798	17.28	16.03
59.304	15.80			(12)	(18)
No. of compar. lines	10	13	Means	+15.68	+16.04
ϵ	±1.07	±1.18	Mean	+15.86	
ϵ_0	±0.31	±0.28	V_a	−12.16	
Ti standards used	4427.266	4399.935	V_d	− 0.21	
	4481.438	4457.600	Reduction to Sun	−12.37	
	4527.490	4512.906	Radial velocity	+ 3.5	

β GEMINORUM—A 426.

1903, April 8, 15h 14m. Hour angle W 2h 46m.

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	F.	A.	Line, Wave-Length in Sun	F.	A.
4427.420	+ 31.77	+ 32.23	4468.663	+ 31.74	+ 32.74
28.711		33.53	76.214	33.90	32.09
35.184	33.47		82.376	35.19	34.39
41.881		31.80	97.046	33.41	34.07
42.510	32.94	33.95	4501.422	31.11	31.84
43.976	33.40	33.47	22.853	34.74	
47.892	32.09	33.23	28.798	32.24	34.04
56.030	32.64	31.96	47.196	32.97	
57.656		35.32	54.211	34.24	
				(15)	(14)
No. of compar. lines	15	10	Means	+ 33.06	+ 33.19
ϵ	± 1.16	± 1.10	Mean	+ 33.12	
ϵ_0	± 0.30	± 0.30	V_a	− 29.42	
Ti standards used	4427.266	4427.266	V_d	− 0.20	
	4481.438	4481.438	Reduction to Sun	− 29.62	
	4552.632	4527.490	Radial velocity	+ 3.5	

α *CRATERIS*—B 491.

1903, February 4, $19^h\ 55^m$. Hour angle W $0^h\ 9^m$.

Star spectrum too weak; comparison spectrum strong.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4457.656		+30.49	4565.750		+31.59
68.663		32.49	71.275		31.10
76.214		31.30	81.634		30.18
97.046		29.15	84.018		29.83
4500.451		29.99	90.126	+29.33	30.58
01.422		32.52	91.614 B		29.52
15.475		31.62	4611.455	27.50	30.04
28.798	+31.45	33.85	25.227	33.78	
46.129	26.64		26.358	27.67	
54.211	29.96		54.743 B	31.44	
60.225		31.70	73.370 B	27.08	
63.939	29.56			(11)	(16)
No. of compar. lines	10	11	Means	+29.44	+31.00
ϵ	±2.18	±1.26	Mean +30.22		
ϵ_0	±0.66	±0.32	V_a +16.66		
Ti standards used	4527.490	4457.600	V_d − 0.01		
	4617.452	4527.490	Reduction to Sun		+16.65
	4682.088	4617.452	Radial velocity		+46.9

α *CRATERIS*—A 438.

1903, April 16, $17^h\ 24^m$. Hour angle W $2^h\ 12^m$.

Star spectrum weak; comparison spectrum strong.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+61.84		4497.046	+60.48	+60.21
41.881		+61.23	4501.422		60.62
42.510		62.91	15.475	59.23	61.69
47.892		62.08	22.853	66.83	
57.656	60.61	61.42	28.798	63.36	62.78
60.460		63.40	47.196	62.44	
68.663		63.01	48.938		62.89
76.214		63.58	54.211	65.71	61.44
82.376	61.75		63.939		60.58
85.846		61.83	65.750		62.12
94.738	57.50				
				(10)	(16)
No. of compar. lines	9	13	Means	+61.98	+61.99
ϵ	±2.82	±1.04	Mean +61.98		
ϵ_0	±0.89	±0.26	V_a −14.52		
Ti standards used	4427.266	4449.313	V_d − 0.18		
	4496.318	4496.318	Reduction to Sun		−14.70
	4555.662	4555.662	Radial velocity		+47.3

α *CRATERIS* — A 444.

1903, April 22, 16 h 56 m. Hour angle W 2 h 6 m.

Star spectrum weak; comparison spectrum strong.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	+69.22		4512.906	+63.53	+65.32
35.184	65.65		28.798	65.68	62.18
45.641		+64.14	44.845 B		62.09
47.892	69.16	65.86	47.196	61.72	
60.460		64.07	63.939		64.91
68.663	67.77	65.42	71.275		62.91
76.214	65.26	63.32	81.634		64.08
82.376	64.29	64.96	86.191 B	67.86	
90.900		63.36	4606.404		64.48
97.046	66.01		11.455		64.37
4501.422	63.94	62.81		(12)	(16)
No. of compar. lines	12	14	Means	+65.84	+64.02
ϵ	± 1.32	± 1.15	Mean	+64.93	
ϵ_0	± 0.67	± 0.30	V_a	−16.73	
Ti standards used	4427.266	4427.266	V_d	−0.17	
	4512.906	4512.906	Reduction to Sun		−16.90
	4590.126	4617.452	Radial velocity		+48.0

α *BOÖTIS*—A 373

1902, September 6, 13h 43m. Hour angle W 4h 38m.

Star spectrum excellent; comparison spectrum excellent.

Line, Wave-Length in Sun	F.	A.	Line, Wave-Length in Sun	F.	A.
4427.420	+10.63	+10.84	4482.376	+11.84	+10.90
28.711		11.92	94.738	10.67	11.81
34.021	13.59		97.046	11.47	10.40
35.851		11.15	4501.422		10.39
42.510	12.28	11.95	12.906	12.03	10.76
43.976	10.32	11.88	15.475		11.69
47.892	12.20	11.93	28.798	11.06	11.26
56.030	11.17	11.24	48.938	13.45	
57.656	11.30		54.211	12.77	
66.701		11.61			
68.663	13.62				
75.026		10.40			
76.214	12.06	11.59		(16)	(17)
No. of compar. lines	17	11	Means	+11.90	+11.28
ϵ	±1.05	±0.57	Mean	+11.59	
ϵ_0	±0.26	±0.14	V_a	−15.79	
Ti standards used	4427.266	4427.266	V_d	−0.30	
	4496.318	4465.975	Reduction to Sun		−16.09
	4552.632	4527.490	Radial velocity		−4.5

α *BOÖTIS*—B 492

1903, February 4, $21^h\ 39^m$. Hour angle E $1^h\ 35^m$.

Star spectrum good; comparison spectrum weak.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−31.02	−29.60	4501.422	−27.31	−26.64
41.881		30.58	12.906	25.98	26.31
42.510	27.61	27.47	26.644		29.41
43.976		28.95	28.798	29.93	30.26
47.892	27.10		46.129	30.60	30.08
56.030	29.06	27.72	48.024		29.66
57.656	29.33	28.72	54.211	29.89	
68.663	26.84	26.64	63.939	29.63	
76.214	28.01	28.61	65.750		28.76
82.376		27.30		(13)	(16)
No. of compar. lines	13	12	Means	−28.64	−28.54
ϵ	±1.59	±1.38	Mean	−28.59	
ϵ_0	±0.44	±0.34	V_a	+24.03	
Ti standards used	4427.266	4427.266	V_d	+ 0.13	
	4501.445	4501.445	Reduction to Sun	+24.16	
	4563.939	4563.939	Radial velocity	− 4.4	

α *BOÖTIS*—B 497.

1903, March 24, $22^h\ 48^m$. Hour angle W $2^h\ 49^m$

Star spectrum very good; comparison spectrum very good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−13.82	−13.81	4512.906	−12.82	−14.09
28.711	13.00		18.198		14.93
41.881		14.78	28.798	13.18	13.97
42.510	13.16	12.29	34.953	14.74	
47.892	14.56	13.55	46.129	16.49	
56.030	15.75	14.53	48.024	13.19	
57.656	14.80	12.44	48.938	11.40	
68.663	13.02	12.41	54.211	14.09	
76.214	14.40	13.53	71.275	15.54	
82.376	13.38	13.05			
94.738	15.14	14.68			
97.046	13.54	12.27			
4501.422	16.79	14.45		(20)	(15)
No. of compar. lines	17	12	Means	−14.14	−13.65
ϵ	±1.35	±0.96	Mean	−13.90	
ϵ_0	±0.30	±0.25	V_a	+8.85	
Ti standards used	4427.266	4427.266	V_d	−0.22	
	4481.438	4481.438	Reduction to Sun	+ 8.63	
	4555.662	4527.490	Radial velocity	− 5.3	

α BOÖTIS—A 433.

1903, April 8, $21^h\ 46^m$. Hour angle W $2^h\ 42^m$.

Star spectrum excellent; comparison spectrum excellent.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	−7.25	−7.45	4501.422	−7.73	−7.46
28.711		9.14	12.063		8.51
41.881		8.24	12.906	3.79	
42.510	7.42	7.36	22.853	4.38	
43.976	6.61	6.88	28.798	4.37	6.89
47.892	8.43	7.21	46.129	8.11	
56.030	8.01	7.27	48.024	5.34	
57.656	7.13	7.27	48.938	8.11	
68.663	5.03	5.90	54.211	6.06	
76.214	6.83	7.84			
82.376	7.09	7.16			
94.738	9.01	7.74			
97.046	7.93	6.47		(19)	(16)
No. of compar. lines	15	12	Means	−6.77	−7.42
ϵ	±1.53	±0.96	Mean	−7.10	
ϵ_0	±0.35	±0.25	V_a	+2.43	
Ti standards used	4427.266	4427.266	V_d	−0.21	
	4496.318	4481.438	Reduction to Sun	+2.22	
	4555.662	4527.490	Radial velocity	−4.9	

α BOÖTIS—B′ 499.

1903, May 6, $14^h\ 44^m$. Hour angle E $2^h\ 25^m$.

Star spectrum excellent; comparison spectrum slightly weak.

4427.420	+3.12	+2.51	4501.422	+2.46	+4.66
41.881		2.77	12.906	5.98	
42.510	5.94	4.72	26.644	5.23	3.91
43.976	6.55	5.40	28.798	6.16	4.70
47.892	3.71	3.37	34.953	2.84	
56.030	4.10	3.70	46.129	3.89	
57.656	3.90	4.10	48.024	4.48	
60.460		4.91	48.938	3.43	
68.663	6.04	4.70	54.211	1.71	
76.214	3.48	2.08	63.939	4.73	
82.376	5.02	4.42	71.275	3.15	
94.738		2.94		(20)	(15)
No. of compar. lines	18	10	Means	+4.30	+3.93
ϵ	±1.37	±1.00	Mean	+4.12	
ϵ_0	±0.31	±0.26	V_a	−9.36	
Ti standards used	4427.266	4427.266	V_d	+0.19	
	4501.445	4481.438	Reduction to Sun	−9.17	
	4572.156	4527.490	Radial velocity	−5.1	

β OPHIUCHI—B 378.

1902, August 7, $15^{h}\ 6^{m}$. Hour angle W $0^{h}\ 37^{m}$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	+ 8.53	+ 8.53	4501.422		+ 8.33
42.510	10.80	10.12	12.906	+ 8.31	9.83
43.976		10.19	17.702		11.55
47.892	10.85	9.24	26.644	9.21	
56.030		11.17	27.518	10.53	
57.656	9.82	9.55	28.798	6.95	10.13
59.922		10.35	46.129	10.42	
68.663	10.67	7.52	48.024	9.76	
76.214	8.04	11.12	48.938	9.29	
82.376	9.43	8.36	54.211	8.49	
94.738		8.01			
97.046	8.94	10.34		(16)	(16)
No. of compar. lines	11	11	Means	+ 9.38	+ 9.65
ϵ	±1.13	±1.21	Mean	+9.52	
ϵ_0	±0.28	±0.30	V_a	−20.20	
Ti standards used	4427.266	4427.266	V_d	− 0.05	
	4481.438	4481.438	Reduction to Sun	−20.25	
	4555.662	4527.490	Radial velocity	−10.7	

β OPHIUCHI—A 451.

1903, April 30, $19^{h}\ 55^{m}$. Hour angle E $1^{h}\ 8^{m}$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	F.	A.	Line, Wave-Length in Sun	F.	A.
4427.420	−29.26	−30.21	4468.663	−29.46	−28.85
35.184	28.34		76.214	30.22	29.48
41.881		31.53	82.376	28.50	29.03
42.510	32.87	28.35	94.738	35.89	30.36
43.976		30.10	96.222 B	27.77	
47.892	31.14	28.58	97.046	27.67	29.61
56.030	29.94	31.36	4501.422	30.77	30.44
57.656	31.35	30.14	28.798	30.46	31.05
60.460	30.39	28.37		(15)	(15)
No. of compar. lines	12	11	Means	−30.27	−29.83
ϵ	±2.11	±1.05	Mean	−30.05	
ϵ_0	±0.54	±0.27	V_a	+18.46	
Ti standards used	4427.266	4427.266	V_d	+ 0.10	
	4481.438	4481.438	Reduction to Sun	+18.56	
	4527.490	4527.490	Radial velocity	− 11.5	

β OPHIUCHI — B′ 508.

1903, June 26, $15^h\ 32^m$. Hour angle E $1^h\ 42^m$.

Star spectrum good; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−9.28	−7.45	4501.422		−6.06
43.976		6.55	12.063		7.44
47.892	5.93	6.00	12.906	−6.64	7.31
56.030	8.41	6.46	17.702		4.45
57.656	6.53	7.27	27.518	5.03	
59.922		6.45	28.798	6.62	6.22
68.663	3.62	4.29	46.129	7.12	
76.214	7.91	6.90	48.024	4.55	
82.376	6.15	6.36	48.938	7.67	
94.738	7.60	5.87	54.221	7.18	
97.046	9.27			(16)	(15)
No. of compar. lines	14	12	Means	−6.84	−6.34
ϵ	±1.58	±0.96	Mean	−6.59	
ϵ_0	±0.39	±0.25	V_a	−4.58	
Ti standards used	4427.266	4427.266	V_d	+0.15	
	4481.438	4481.438	Reduction to Sun	−4.43	
	4555.662	4527.490	Radial velocity	−11.0	

γ AQUILAE — B 398.

1902, August 27, $17^h\ 2^m$. Hour angle W $1^h\ 50^m$.

Star spectrum weak; comparison spectrum too strong.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4456.030	+12.38		4528.798	+13.97	
57.656	14.53		46.129	14.57	+14.51
68.663	15.10	+14.43	48.024	15.62	
76.214	12.19	11.19	48.938		10.09
82.376		10.04	54.211	12.25	
85.846		10.83	60.225		13.55
97.046	13.34	14.27	65.750		14.25
4500.451		14.39	74.899		13.44
01.422	12.32		4603.126		12.31
12.906	13.22	11.50	11.455		13.07
15.475	13.28				
27.518	14.57	14.50		(13)	(15)
No. of compar. lines	13	15	Means	+13.64	+12.82
ϵ	±1.17	±1.70	Mean	+13.23	
ϵ_0	±0.32	±0.44	V_a	−14.49	
Ti standards used	4455.485	4481.438	V_d	−0.16	
	4497.023[1]	4548.938	Reduction to Sun	−14.65	
	4555.662	4617.452	Radial velocity	−1.4	

[1] *Cr.*

γ *AQUILAE* — B 417.

1902, October 9, 14^h 12^m. Hour angle W 1^h 52^m.

Star spectrum slightly weak; comparison spectrum good.

Line, Wave Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+22.28		4501.422		+20.05
28.711		+21.53	12.906	+22.39	22.46
34.021	24.01		28.798	23.64	22.98
41.881		21.67	46.129		23.61
42.510	23.08	21.60	47.196	23.14	
47.892	23.12	22.99	48.938		21.75
56.030	22.61	21.33	56.202 B		22.18
68.663	26.64		60.225		21.77
76.214	22.24	22.91	63.939		21.42
82.376	23.15	23.15	65.750		21.28
97.046	23.80	23.60			
				(12)	(17)
No. of compar. lines	13	14	Means	+23.34	+22.13
ϵ	±1.19	±0.96	Mean	+22.74	
ϵ_0	±0.34	±0.23	V_a	−24.85	
Ti standards used	4427.266	4427.266	V_d	− 0.16	
	4481.438	4481.438	Reduction to Sun		−25.01
	4544.864	4555.662	Radial velocity		− 2.3

γ *AQUILAE* — B′ 509.

1903, June 26, 17^h 25^m. Hour angle E 1^h 50^m.

Star spectrum good; comparison spectrum good.

Line, Wave Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−11.85		4512.906	−13.56	12.76
29.366	12.58		17.702		12.34
42.510	9.99		26.644		13.58
43.976	12.02		28.798	11.32	12.05
47.892	12.40		46.129	12.99	12.33
56.030	14.06		48.024	13.71	12.99
57.656	11.50		48.938	12.20	12.85
68.663	8.33	−11.00	54.211	12.90	
76.214	13.80	12.66	63.939	12.02	10.51
82.376	14.58	13.31	71.275	15.68	14.76
97.046	11.80	11.94	77.356		14.54
4501.422	12.92	13.79			
12.063		−15.02			
				(20)	(16)
No. of compar. lines	20	13	Means	−12.51	−12.90
ϵ	±1.57	±1.28	Mean	−12.70	
ϵ_0	±0.34	±0.32	V_a	+10.70	
Ti standards used	4427.266	4465.975	V_d	+ 0.15	
	4496.318	4527.490	Reduction to Sun		+10.85
	4590.126	4590.126	Radial velocity		− 1.8

ε PEGASI—A 364.

1902, July 31, 19^{h} 15^{m}. Hour angle W 0^{h} 14^{m}.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	−3.52		4497.046	−4.53	−5.20
34.021	4.67		4500.451		2.27
42.510	2.57		01.422	2.93	4.26
45.641		−4.38	05.003		3.86
47.892	2.97		12.906	3.26	3.06
49.313		4.78	27.518		5.30
56.030	5.99		28.798	3.64	3.11
57.656	4.91		46.129	5.28	5.14
59.922	4.24		48.938	6.00	5.47
68.663	1.14	2.35	65.750		5.45
76.214	4.02	5.76	74.899		2.43
82.376		2.81	90.126		3.85
88.363 B	4.21			(16)	(17)
No. of compar. lines	17	14	Means	−3.99	−4.09
ϵ	±1.28	±1.22	Mean	−4.04	
ϵ_0	±0.32	±0.30	V_a	+10.17	
Ti standards used	4427.266	4449.313	V_d	−0.02	
	4481.438	4512.906	Reduction to Sun	+10.15	
	4544.864	4590.126	Radial velocity	+6.1	

ε PEGASI—B 379.

1902, August 7, 16^{h} 37^{m}. Hour angle E 1^{h} 55^{m}.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	F.	A.	Line, Wave-Length in Sun	F.	A.
4427.420	+0.47		4512.906	−2.26	−2.46
42.510	+0.67	+0.74	15.475	−3.85	−2.46
47.892	−0.34	−1.08	28.978	−1.26	
56.030	−1.61		34.168		−1.39
57.656	−1.88		46.129	−2.37	−1.58
68.663	0.00	−0.13	48.024	−0.33	
76.214	−1.68	−1.41	48.938	−2.24	−0.73
82.376	−3.01	+0.27	63.939	−0.20	−0.85
97.046		−0.53	65.750		−2.43
97.842	−0.20		71.275	−0.98	−2.76
4501.422	−1.73	−0.47		(18)	(15)
No. of compar. lines	17	13	Means	−1.27	−1.15
ϵ	±1.24	±1.06	Mean	−1.21	
ϵ_0	±0.29	±0.27	V_a	+7.17	
Ti standards used	4427.266	4443.976	V_d	+0.16	
	4481.438	4512.906	Reduction to Sun	+7.33	
	4544.864	4572.156	Radial velocity	+6.1	

ϵ *PEGASI* — B 418.

1902, October 9, $16^h\ 14^m$. Hour angle W $1^h\ 54^m$.

Star spectrum a trifle faint; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4442.510		+27.88	4527.518	+27.82	+26.82
57.656	+26.64		28.798	26.29	
59.922	26.09		34.168		27.44
60.460		26.62	44.077 B		24.81
68.663	28.72	28.58	46.129	25.65	27.64
76.214	27.87	28.74	47.196	25.51	
90.900		25.36	48.024	27.23	
95.664 B		25.15	48.938	26.96	27.29
4501.422		24.98	65.750		26.14
12.906	24.85	25.52		(11)	(14)
No. of compar. lines	12	15	Means	+26.69	+26.64
ϵ	±1.16	±1.34	Mean	+26.66	
ϵ_0	±0.34	±0.36	V_a	−19.98	
Ti standards used	4457.600	4449.313	V_d	−0.16	
	4512.906	4512.906	Reduction to Sun	−20.14	
	4552.632	4555.662	Radial velocity	+ 6.5	

γ *PISCIUM* — B 381.

1902, August 7, $19^h\ 23^m$. Hour angle E $0^h\ 40^m$.

Star spectrum weak; comparison spectrum fair.

4427.420	−28.85		4497.046	−28.07	−29.54
42.510	25.25		4501.422	26.64	27.04
43.976		−29.22	08.455		28.40
47.892	26.90	27.30	12.906	26.58	
56.030	25.98	27.59	15.475	29.55	
57.656	30.94	29.67	26.644		27.69
68.663	27.24	29.26	28.798	28.34	27.75
76.214	25.80	29.95	46.129	26.91	
82.376	27.23	27.43	48.024	27.88	28.02
94.738	27.62	29.02	54.211		29.30
				(16)	(15)
No. of compar. lines	13	11	Means	−27.55	−28.48
ϵ	±1.41	±0.99	Mean	−28.02	
ϵ_0	±0.35	±0.26	V_a	+16.59	
Ti standards used	4427.266	4449.313	V_d	+ 0.06	
	4481.438	4501.445	Reduction to Sun	+16.65	
	4544.864	4552.632	Radial velocity	−11.4	

γ *PISCIUM*—B 415.

1902, October 8, $14^h\ 38^m$. Hour angle E $1^h\ 21^m$.

Star spectrum weak; comparison spectrum good.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4441.881		+0.14	4526.644	+2.52	+2.98
42.510		+3.44	28.798	+6.42	+4.17
47.892		+0.88	46.129	+1.91	+4.35
56.030		−1.21	48.938		+1.38
57.656	+2.15		54.211	+1.32	
59.304	+1.82		58.827	+0.07	
68.663	+4.16	+2.15	65.750		−1.12
76.214	+2.34	+2.21	71.275		−0.98
82.376	+2.14	+2.94	72.156	−2.56	−2.16
94.738	+1.33				
97.046	+4.00	+3.87	81.634		+3.73
4501.422	+3.20	+4.06	90.126	−1.31	
4512.906	+3.12			(16)	(17)
No. of compar. lines	12	13	Means	+2.20	+1.81
ϵ	±1.94	±2.19	Mean	+2.00	
ϵ_0	±0.48	±0.53	V_a	−12.85	
Ti standard used	4457.600	4449.313	V_d	+ 0.12	
	4527.490	4527.490	Reduction to Sun		−12.73
	4590.126	4590.126	Radial velocity		−10.7

γ *PISCIUM*—B 436.

1902, October 30, $14^h\ 39^m$. Hour angle W $0^h\ 5^m$.

Star spectrum very good; comparison spectrum a trifle strong.

Line, Wave-Length in Sun	F.	A.	Line, Wave-Length in Sun	F.	A.
4427.420	+10.50	+11.38	4468.663		+12.48
33.390		12.37	76.214	+11.12	10.79
34.021	11.36		82.376	12.18	11.11
35.851	10.00	8.86	94.738	9.54	9.54
42.510	13.03	12.49	97.046	11.14	10.94
43.976		11.27	4501.422	12.92	11.79
47.892	11.80	11.93	08.455		11.11
54.993 B	9.69		12.906	11.23	
56.030		11.57	15.475	11.62	
57.656	10.56	10.70	28.798	11.59	12.05
66.701	11.00			(16)	(16)
No. of compar. lines	12	12	Means	+11.20	+11.27
ϵ	±1.01	±1.00	Mean	+11.24	
ϵ_0	±0.25	±0.25	V_a	−21.94	
Ti standard used	4427.266	4427.266	V_d	− 0.01	
	4481.438	4481.438	Reduction to Sun		−21.95
	4527.490	4527.490	Radial velocity		−10.7

SUPPLEMENTARY STARS.

ι *AURIGAE*—B 446.

1902, November 6, 15^{h} 30^{m}. Hour angle E 4^{h} 12^{m}.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+2.30		4497.046	+3.40	+4.40
29.366		+2.23	4501.422	2.86	3.40
42.510	5.80	5.06	12.906	2.86	3.52
47.892		4.04	18.198		4.45
56.030	4.17	2.09	26.644	1.32	
57.656	2.35		28.798	3.58	4.30
60.460		4.03	46.129	2.70	
68.663	8.25		48.024	4.81	
76.214	3.08	2.61			
82.376	3.55	3.75		(14)	(12)
No. of compar. lines	12	13	Means	+3.64	+3.66
ϵ	±1.73	±0.93	Mean	+3.65	
ϵ_0	±0.46	±0.27	V_a	+15.07	
Ti standards used	4427.266	4427.266	V_d	+ 0.26	
	4481.438	4481.438	Reduction to Sun	+15.33	
	4544.864	4527.490	Radial velocity	+19.0	

ε *LEONIS*—B 483.

1903, January 8, 21^{h} 56^{m}. Hour angle W 1^{h} 36^{m}.

Star spectrum good; comparison spectrum fair.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	−9.96	−11.65	4494.738	−12.07	−10.94
35.851	14.20	12.03	97.046	10.67	
41.881		8.71	4501.422	9.33	8.33
42.510		9.38	08.455	10.04	10.31
43.976	8.17		15.475	9.83	
47.892	10.38	11.32	28.798	10.06	9.54
57.656	8.14	8.01	46.129		10.88
68.663	9.73	9.26	48.938		11.60
76.214	10.79	10.45	54.211		8.23
82.376	9.43	9.30		(14)	(16)
No. of compar. lines	12	11	Means	−10.20	−10.00
ϵ	±1.53	±1.32	Mean	−10.10	
ϵ_0	±0.41	±0.33	V_a	+15.57	
Ti standards used	4427.266	4427.266	V_d	− 0.13	
	4481.438	4481.438	Reduction to Sun	+15.44	
	4527.490	4555.662	Radial velocity	+ 5.3	

ϵ *LEONIS*— A 427.

1903, April 8, $16^h\ 11^m$. Hour angle W $1^h\ 45^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4399.903		+29.92	4476.214		+28.68
4427.420	+28.51	30.61	82.376	+26.22	27.43
28.711		28.24	97.046	29.67	30.67
41.881		27.34	4501.422	30.64	31.77
42.510		30.17	08.455	31.73	
43.976	29.28	30.10	15.475	30.48	
47.892	31.08	31.41	28.798	32.38	30.59
57.656	26.10	27.18	54.211	32.06	
60.460		28.18			
68.663	29.45	29.52		(13)	(15)
No. of compar. lines	13	11	Means	+29.69	+29.45
ϵ	±2.01	±1.51	Mean	+29.57	
ϵ_0	±0.56	±0.39	V_a	−24.79	
Ti standards used	4427.266	4399.935	V_d	− 0.14	
	4481.438	4457.600	Reduction to Sun	−24.93	
	4552.632	4527.490	Radial velocity	+ 4.6	

ϵ *LEONIS*— A 443.

1903, April 22, $14^h\ 48^m$. Hour angle W $1^h\ 14^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4427.420	+32.85	+32.04	4482.376	+35.99	+35.59
42.510		33.28	94.738		32.35
43.976		36.03	97.046	37.07	
47.892	33.23	35.73	4501.422		33.44
56.030	35.19	35.13	08.455	32.66	34.85
57.656	33.50	33.77	15.475	33.07	
59.304	34.16		28.798	34.56	32.25
60.460		32.34	47.196	33.77	
68.663	34.22	35.23	54.211	34.63	
76.214	34.24	36.05		(14)	(14)
No. of compar. lines	14	11	Means	+34.22	+34.15
ϵ	±1.23	±1.53	Mean	+34.18	
ϵ_0	±0.33	±0.41	V_a	−27.67	
Ti standards used	4427.266	4427.266	V_d	− 0.10	
	4481.438	4481.438	Reduction to Sun	−27.77	
	4555.662	4527.490	Radial velocity	+ 6.4	

γ CEPHEI—B 428.

1902, October 16, $17^h\ 44^m$. Hour angle, W $1^h\ 50^m$.

Star spectrum weak; comparison spectrum good.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4456.030		−49.05	4526.644	−46.57	
57.656	−46.55	49.51	27.518	46.23	
60.460		49.28	28.798	46.68	−46.95
68.663	48.85	49.32	46.129	43.99	47.69
76.214	49.71	49.19	48.024		48.66
82.376	47.70	48.50	48.938		47.59
97.046	48.01		54.211	50.96	
4501.422		49.76	65.750		49.32
12.906	51.03		71.275		48.61
17.702		46.80	72.156		48.20
				(11)	(15)
No. of compar. lines	9	11	Means	−47.87	−48.56
ϵ	±2.20	±0.93	Mean	−48.22	
ϵ_0	±0.66	±0.24	V_a	+7.32	
Ti standards used	4457.600	4455.485	V_d	−0.04	
	4512.906	4501.445	Reduction to Sun	+ 7.28	
	4552.632	4572.156	Radial velocity	−40.9	

γ CEPHEI—A 424.

1903, April 3, $15^h\ 41^m$ Hour angle W $10^h\ 54^m$

Star spectrum good; comparison spectrum a trifle strong.

Line, Wave-Length in Sun	Velocity F.	Velocity A.	Line, Wave-Length in Sun	Velocity F.	Velocity A.
4400.577 B	−31.89		4459.304	−30.53	
08.549	33.05		60.460		−32.67
27.420	32.31	−31.97	68.663	30.26	32.41
28.711		30.74	76.214	32.36	32.96
35.184	32.46		82.376	30.91	30.84
42.510		33.14	94.738	32.29	31.43
43.976		33.87	97.046		32.54
47.892	32.63	30.60	4528.798	32.91	32.65
56.030		33.17	47.196	31.00	
57.656	31.75	31.01	54.211	28.51	
				(14)	(14)
No. of compar. lines	13	10	Means	−31.63	−32.14
ϵ	±1.26	±1.05	Mean	−31.88	
ϵ_0	±0.34	±0.28	V_a	−9.23	
Ti standards used	4399.935	4427.266	V_d	−0.02	
	4481.438	4481.438	Reduction to Sun	− 9.25	
	4555.662	4527.490	Radial velocity	−41.1	

γ *CEPHEI*—B′ 501.

1903, May 6, $17^h\ 42^m$. Hour angle E $8^h\ 50^m$.

Star spectrum good; comparison spectrum good.

Line, Wave-Length in Sun	Velocity		Line, Wave-Length in Sun	Velocity	
	F.	A.		F.	A.
4427.420	−38.06	−39.42	4482.376	−40.27	−39.00
28.711		38.59	94.738		37.56
42.510	37.12	38.20	97.046	38.47	39.01
43.976		38.67	4501.422	41.63	
47.892	39.64	39.50	17.702		38.16
56.030	39.16	39.43	22.853	39.91	
57.656	37.60	38.01	28.798	37.41	37.75
60.460		38.25	46.129	39.37	
68.663	36.30	37.84	48.938	38.10	
76.214	37.86	38.32	54.211	38.45	
				(15)	(15)
No. of compar. lines	13	12	Means	−38.62	−38.51
ϵ	±1.38	±0.64	Mean	−38.56	
ϵ_0	±0.36	±0.16	V_a	−3.06	
Ti standards used	4427.266	4427.266	V_d	+0.06	
	4481.438	4481.438	Reduction to Sun	− 3.00	
	4555.662	4527.490	Radial velocity	−41.6	

SUMMARY.

The results of the above detailed reductions are summarized in the following table. The values obtained by the two observers are placed in parallel columns, followed by the difference between the two for each plate. In the last column is given the mean of both determinations, and, finally, at the foot of the summary for each star are found the results derived from all of the plates.

In the final determinations given, those for β *Leporis* and ι *Aurigae* are, of course, entitled to low weight, since but one plate has been measured in the case of each star. Among the other stars of the list, the result obtained for α *Crateris* is probably subject to the greatest uncertainty: the low altitude and photographic faintness of the star have made it a difficult object, and all of the plates secured have been too weak for the most accurate measurement. A few of the plates of γ *Aquilae* and γ *Piscium* are also somewhat weak, but in a less degree.

The excellent agreement of the values of the two observers for each plate of ϵ *Leonis*, but unusually large discordance in the

results given by the different plates, might lead one to suspect that the range of 1.8 km. may be real, but a much larger number of plates would be required before a definite conclusion could be reached on the subject. The range in the values obtained for *a Persei* we consider to be due to the character of its spectrum.

α ARIETIS.

Series and Number	Date	F.	A.	F.-A.	Mean
B 420	1902, October 9	−13.23	−13.40	+0.17	−13.3
B 430	October 29	−13.38	−13.99	+0.61	13.7
B 465	November 27	−14.01	−14.12	+0.11	14.1
	1902.84	−13.5	−13.9	+0.30	−13.7

α PERSEI.

Series and Number	Date	F.	A.	F.-A.	Mean
B 382	1902, August 7	−2.14	−1.45	−0.69	−1.8
B 431	October 29	−1.63	−1.69	+0.06	1.7
B 458	November 19	−3.18	−2.79	−0.39	3.0
	1902.77	−2.3	−2.0	−0.34	−2.1

β LEPORIS.

Series and Number	Date	F.	A.	F.-A.	Mean
B 449	1902, November 6	−12.20	−12.55	+0.35	−12.4
	1902.85	−12.2	−12.6	+0.35	−12.4

β GEMINORUM.

Series and Number	Date	F.	A.	F.-A.	Mean
B 477	1902, December 31	+2.83	+3.77	−0.94	+3.3
A 398	1903, February 5	+3.31	+3.67	−0.36	3.5
A 426	April 8	+3.44	+3.57	−0.13	3.5
	1903.12	+3.2	+3.7	−0.48	+3.4

α CRATERIS.

Series and Number	Date	F.	A.	F.-A.	Mean
B 491	1903, February 4	+46.09	+47.65	−1.56	+46.9
A 438	April 16	+47.28	+47.29	−0.01	47.3
A 444	April 22	+48.94	+47.12	+1.82	48.0
	1903.23	+47.4	+47.4	+0.08	+47.4

α *BOÖTIS.*

Series and Number	Date	F.	A.	F.-A.	Mean
A 373	1902, September 6	−4.19	−4.81	+0.62	−4.5
B 492	1903, February 4	−4.48	−4.38	−0.10	4.4
B 497	March 24	−5.51	−5.02	−0.49	5.3
A 433	April 8	−4.55	−5.20	+0.65	4.9
B′ 499	May 6	−4.87	−5.24	+0.37	5.1
	1903.12	−4.7	−4.9	+0.22	−4.8

β *OPHIUCHI.*

B 378	1902, August 7	−10.87	−10.60	−0.27	−10.7
A 451	1903, April 30	−11.71	−11.27	−0.44	11.5
B′ 508	June 26	−11.27	−10.77	−0.50	11.0
	1903.14	−11.3	−10.9	−0.40	−11.1

γ *AQUILAE.*

B 398	1902, August 27	−1.01	−1.83	+0.82	−1.4
B 417	October 19	−1.67	−2.88	+1.21	2.3
B′ 509	1903, June 26	−1.66	−2.05	+0.39	1.8
	1902.98	−1.4	−2.2	+0.81	−1.8

ε *PEGASI.*

A 364	1902, July 31	+6.16	+6.06	+0.10	+6.1
B 379	August 7	+6.06	+6.18	−0.12	6.1
B 418	October 9	+6.55	+6.50	+0.05	6.5
	1902.65	+6.2	+6.2	+0.01	+6.2

γ *PISCIUM.*

B 381	1902, August 7	−10.90	−11.83	+0.93	−11.4
B 415	October 8	−10.53	−10.92	+0.39	10.7
B 436	October 30	−10.75	−10.68	−0.07	10.7
	1902.73	−10.7	−11.1	+0.42	−10.9

SUPPLEMENTARY STARS.

ι AURIGAE.

Series and Number	Date	F.	A.	F.-A.	Mean
B 446	1902, November 16	+18.97	+18.99	−0.02	+19.0
	1902.85	+19.0	+19.0	−0.02	+19.0

ε LEONIS.

Series and Number	Date	F.	A.	F.-A.	Mean
B 483	1903, January 8	+5.24	+5.44	−0.20	+5.3
A 427	April 8	+4.76	+4.52	+0.24	4.6
A 443	April 22	+6.45	+6.38	+0.07	6.4
	1903.20	+5.5	+5.4	+0.04	+5.5

γ CEPHEI.

Series and Number	Date	F.	A.	F.-A.	Mean
B 428	1902, October 16	−40.59	−41.28	+0.69	−40.9
A 424	1903, April 3	−40.88	−41.39	+0.51	41.1
B' 501	May 6	−41.62	−41.51	−0.11	41.6
	1903.13	−41.0	−41.4	+0.36	−41.2

An examination of the above results for systematic differences between the two observers indicates a slight tendency for F.−A. to be positive. A general mean for all of the stellar plates (37) gives F.−A. a little less than +0.10 kilometers. It is, however, extremely doubtful whether this is to be considered as other than accidental, as it is chiefly due to the abnormally large positive differences given by two somewhat inferior plates of *γ Aquilae*. Accordingly it would seem that the personality errors are pretty evenly balanced in this series of measures; particularly when the possibility is considered of errors due to the regular difference in the corrections applied for curvature.

No evidence is shown in the results given above of any dependence of the value obtained upon the position of the spectrograph in reference to the pier of the telescope. Of other instrumental causes which might give rise to errors, temperature changes may be regarded as eliminated, as a brief examination

of the Journal of Observations will show. Possible errors arising from changes in the electrical conditions of the comparison spectrum apparatus may also be disregarded, as this has remained without essential modification during the entire interval covered by the observations.

Spectrographic determinations of the radial velocities of four of the stars in the principal list, and of one of the supplementary list, have been published since Vogel and Scheiner obtained their results in 1889–90. These are collected in the following table. For convenience in comparison, the value we have found for each star in this paper is repeated immediately below its name.

Star	Observer	Velocity	Epoch	No. of Plates	Range	Reference
		km			km	
α Arietis −13.7	Campbell	−14.1	1896.8	4	0.6	ASTROPHYSICAL JOURNAL, **8**, 150, 1898
	Adams	−13.7	1901.9	1	...	ASTROPHYSICAL JOURNAL, **15**, 24, 1902
	Newall	−14.3	1902.8	3	2.8	*Monthly Notices*, **63**, 298, 1903
α Persei −2.1	Campbell	− 2.4	1897.8	4	1.7	ASTROPHYSICAL JOURNAL, **8**, 150, 1898
	Vogel	− 3.2	1901.0	13	3.3	ASTROPHYSICAL JOURNAL, **13**, 322, 1901
	Newall	− 2.6	1902.8	14	5.7	*Monthly Notices*, **63**, 298, 1903
α Boötis −4.8	Frost and Adams	− 4.3	1902.3	8	1.8	*Publications of the Yerkes Observatory*, **2**, 35, 1903
	Newall	− 5.8	1903.4	5	2.7	*Monthly Notices*, **63**, 298, 1903
ε Pegasi +6.2	Campbell	+ 5.7	1897.8	4	1.2	ASTROPHYSICAL JOURNAL, **8**, 150, 1898
ε Leonis +5.5	Wright	+ 5.1	1899.4	7	2.7	ASTROPHYSICAL JOURNAL, **11**, 414, 1900
	Adams	+ 4.0	1900.8	3	0.9	ASTROPHYSICAL JOURNAL, **15**, 25, 1902

To us a surprising difference in the above comparison is that of 0.5 km between our earlier measures of eight plates of *α Boötis* and the five plates of this paper. We attach greater weight to the present series, on account of the improved method of insuring the uniform illumination of the collimator lens by the light from the spark. But the range in velocity observed in either series is larger than might be expected for a star having such well-defined lines.

There would appear to be a slight tendency toward a systematic difference between our results and those of other observers, in the direction of a larger positive, or smaller negative, value for our velocities. The amount is so small, however, and the quantity of material with which comparison may be made is at present so meager, that no certain inference can now be drawn. An

investigation of this and of many other matters of interest will be possible as the results of other participators in the co-operation are published. It is known that, owing to changes being made in their spectrographs, at least two of those planning to co-operate have been unable to carry out the program during the past year, but expect to do so during the present year. A general comparison of approximately contemporaneous results by all participators will therefore not be available until later, but we have thought that the prompt publication of our results would nevertheless be of service. We shall continue our observations this year, following the program as closely as possible.

YERKES OBSERVATORY,
August 27, 1903.

ON THE SPECTRA OF IMPERFECT GRATINGS.

By A. A. Michelson.

It will be convenient to consider separately gratings which act by opacity and those depending on retardation. The simplest case of the former may be represented by the expression

$$y = a \cos ks + b \sin ks \text{ ,}^{1} \qquad (1)$$

in which y represents the reflection (or transmission) factor at any point s of a line perpendicular to the rulings, and $k = 2\pi/\sigma$, where σ is the constant grating-space.

If a plane wave-train of frequency $n/2\pi$ fall normally on such a grating, the effect in a direction making an angle θ with the normal will be

$$\int y ds \cos (nt - \partial)$$

where

$$\partial = \frac{2\pi}{\lambda} s \sin \theta \ .$$

The intensity of the diffracted light is

$$I = \left[\int y ds \cos \partial\right]^2 + \left[\int y ds \sin \partial\right]^2 \qquad (2)$$

or

$$I = a^2 \left[\int \cos ks \cos \partial \, ds\right]^2 + b^2 \left[\int \sin ks \sin \partial \, ds\right]^2 .$$

The integrals, taken from $-\infty$ to ∞, have sensible values only at $\sin \theta = \pm \frac{\lambda}{\sigma}$, and the intensity in these directions is

$$I = a^2 + b^2 \ .$$

If the grating cannot be represented by a simple sine curve, let

$$y = \Sigma a_m \cos mks + \Sigma b_m \sin mks \ .$$

This value substituted in (2) gives

$$I_m = a_m^2 + b_m^2$$

in directions such that $\sin \theta = \pm \frac{m\lambda}{\sigma}$.

[1] To avoid negative values, a constant should be added, but this affects only the central image and not the lateral spectra.

As an illustration suppose $y = 1$ from $s = 0$ to $s = \frac{1}{2}\sigma$, and $y = 0$ from $s = \frac{1}{2}\sigma$ to $s = \sigma$.

The value of the coefficients in the series for y will then be

$$a_m = 0 \ , \quad b_0 = \frac{\pi}{2} \ , \quad b_1 = 1 \ , \quad b_2 = 0 \ , \quad b_3 = \frac{1}{3} \ , \quad \text{etc.}$$

The intensities of the spectra will be in the proportion of $\frac{\pi^2}{4}$ for the central image and $1, 0, \frac{1}{3^2}, 0, \frac{1}{5^2}$, etc., for the spectra in their order, in agreement with the results obtained by Lord Rayleigh.[1]

In the case of a reflection grating the retardation is

$$\partial = \frac{2\pi}{\lambda}[s \sin\theta - y\,(1 + \cos\theta)]\,,^*$$

and the effect in a direction θ will be

$$\int ds \cos(nt - \partial) \ . \tag{3}$$

The intensity will be

$$I = \left[\int ds \cos\partial\right]^2 + \left[\int ds \sin\partial\right]^2 . \tag{4}$$

Putting

$$\cos\frac{2\pi}{\lambda}\left(y \cdot 2\cos^2\frac{\theta}{2}\right) = \Sigma a_m \cos mks \ ,$$

and

$$\sin\frac{2\pi}{\lambda}\left(y \cdot 2\cos^2\frac{\theta}{2}\right) = \Sigma b_n \sin mks \ ,$$

also

$$\frac{2\pi}{\lambda}\left(m\frac{\lambda}{\sigma} - \sin\theta\right) = p \ ,$$

and

$$\frac{2\pi}{\lambda}\left(m\frac{\lambda}{\sigma} + \sin\theta\right) = p' \ ,$$

$$\sqrt{I} = \sum a_m \frac{\sin ps}{p} + \sum a_m \frac{\sin p's}{p'} + \sum b_m \frac{\sin ps}{p} - \sum b_m \frac{\sin p's}{p'} \ .$$

[1] *Lord Rayleigh Scientific Papers*, Vol. III.

*For a transmission grating $\partial = \frac{2\pi}{\lambda}[sn \sin\theta + y\,(1 - n\cos\theta)]$.

The limits being infinite, I has finite values only at $p=0$ and at $p'=0$, or for $\sin\theta=m\lambda/\sigma$ and $\sin\theta=-m\lambda/\sigma$. For the former

$$I=(a_m+b_m)^2$$

and for the latter

$$I'=(a_m-b_m)^2 \ . \tag{5}$$

If $a_m=b_m$, all the negative spectra vanish. If of the positive spectra all are to vanish except the mth, then

$$\cos\frac{2\pi}{\lambda}\left(y\,.\,2\cos^2\frac{\theta}{2}\right)=\cos mks \ ,$$

$$\sin\frac{2\pi}{\lambda}\left(y\,.\,2\cos^2\frac{\theta}{2}\right)=\sin mks \ ,$$

which gives

$$y=(x+n\sigma)\tan\frac{1}{2}\theta \ . \tag{6}$$

The section of the grating surface is that represented in Fig. 1. It is essentially the arrangement of plates in a reflecting echelon.

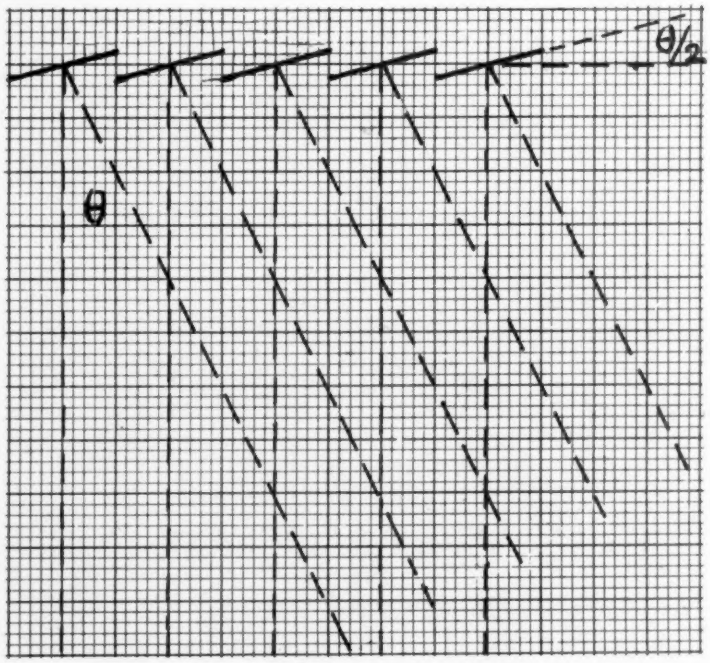


Fig. 1.

In actual gratings the ruled space is limited, and the spectrum of a homogeneous source will not be a line, but an image of finite width. It is proposed to find the distribution of light in this image for any given grating. This distribution can be affected only by the imperfections of the ruling (*e. g.* changes in the strength of the lines or in their spacing), but not by the character of the furrow. We may therefore let

$$y = \phi(s) \cos mks + \psi(s) \sin mks = f(s) \ .$$

By Fourier's formula

$$f(s) = \int_{-\infty}^{\infty} (C \cos asda + S \sin asda) \ ,$$

where
$$C = \int_{-\infty}^{\infty} f(\lambda) \cos a\lambda d\lambda \ ,$$

and
$$S = \int_{-\infty}^{\infty} f(\lambda) \sin a\lambda d\lambda \ . \tag{7}$$

A single element of $f(s)$ is therefore

$$C \cos as + S \sin as \ , \tag{7'}$$

which gives a spectrum in a direction

$$\sin\theta = \pm ca \quad \text{whose intensity is} \quad I = C^2 + S^2 \ . \tag{8}$$

If the spacing is exact, $\psi = C\phi$, whence

$$I = \left[\int \phi(\lambda) \cos p\lambda d\lambda\right]^2 + \left[\int \phi(\lambda) \sin p\lambda d\lambda\right]^2 ,$$

where $p = mk \pm a$. The spectrum images are always symmetrical.

If the spacing is not exact, but the strength of the rulings is constant, the usual case in actual gratings,

$$f(s) = \cos(ks - \omega) \ ,$$

$$\phi = \cos\omega \ , \quad \psi = \sin\omega \ , \quad C = \int \cos(\omega - p\lambda) d\lambda \ ,$$

$$S = \int \sin(\omega - p\lambda) d\lambda \ ;$$

hence

$$I = \left[\int \cos(\omega - p\lambda) d\lambda\right]^2 + \left[\int \sin(\omega - p\lambda) d\lambda\right]^2 . \tag{9}$$

The spectral images are generally unsymmetrical.

One form of spacing error which doubtless occurs in practice is that caused by the sudden release of strain in some part of

the course, thus producing a "slip" in all the subsequent rulings. Such a case may be represented by putting $\omega = -a$ from $-l$ to 0 and $\omega = a$ from 0 to l. These values substituted in (9) give

$$C = \frac{1}{p}(\sin a - \sin(a - pl)) \; . \tag{10}$$

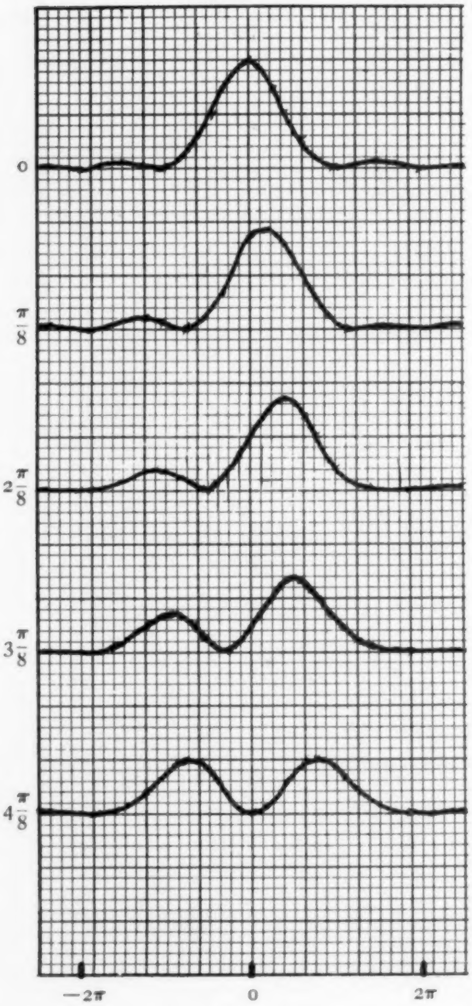


FIG. 2.

The intensity curve which is given by squaring this expression is represented in Fig. 2 for $a = 0, \frac{1}{8}\pi, \frac{2}{8}\pi, \frac{3}{8}\pi$, and $\frac{4}{8}\pi$.

The minima are readily found by putting $C = 0$, which gives

$$pl = 2\left(a - \frac{2n+1}{2}\pi\right) \; . \tag{11}$$

The maxima may be obtained by differentiating (10) for p and equating to zero. It can be shown, however, that the result is given with an error less than 0.002π from $a = \pi/2$ to $a = -\pi/2$ by the expression

$$p_1 l = \frac{3}{2}(a - n\pi) \; . \tag{12}$$

If h is the actual linear displacement of the rulings, $a = 2\pi h/\sigma$. Substituting this value in (12), restoring $a = k - p$ and remembering that $\sin\theta = \frac{\lambda}{2\pi}a$, we have

$$\sin\theta = \frac{\lambda}{\sigma} - \frac{3}{2}\frac{\lambda}{l}\frac{h}{\sigma} + \frac{3}{4}\frac{n\lambda}{l} \; .$$

For $h=0$ and $n=0$, $\sin\theta_0=\lambda/\sigma$, whence

$$\sin\theta-\sin\theta_0=\cos\theta\partial\theta=\frac{\lambda}{\sigma}\left(\frac{3h}{2l}-\frac{3}{4}\frac{n\sigma}{l}\right),$$

$$\frac{\partial\lambda}{\lambda}=\frac{\partial\theta}{\tan\theta}=\frac{3}{2}\frac{h}{l}-\frac{3}{4}\frac{n\sigma}{l}.$$

If the observations are made in the mth spectrum, σ must be replaced by σ/m. Putting also $2l=N\sigma$ we have

$$\frac{\partial\lambda}{\lambda}=\frac{3}{N}\left(\frac{h}{\sigma}-\frac{n}{2m}\right).$$

Treating (11) in a similar way,

$$\frac{\partial_1\lambda}{\lambda}=\frac{4}{N}\left(\frac{h}{\sigma}-\frac{2n+1}{4m}\right).$$

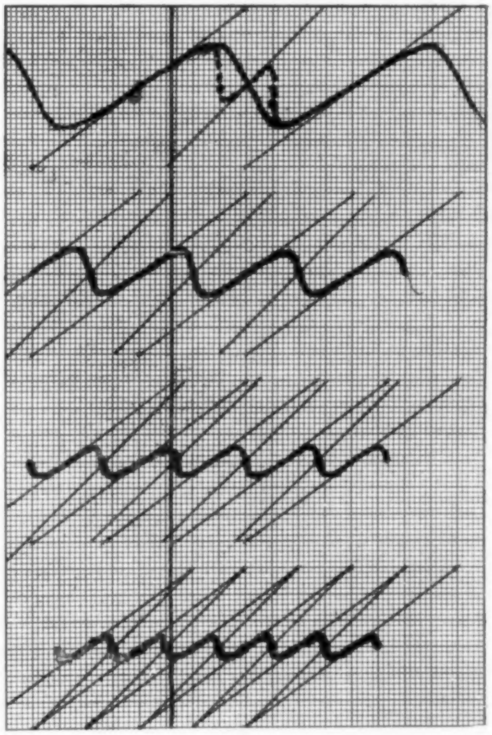


FIG. 3.

These expressions represent the percentage error due to setting on the maximum and the minimum respectively. The graphs of these errors are represented in Fig. 3 by the systems of straight lines.

The actual phase which may be selected for measurement will, however, depend largely on the observer. When h is small, this will be the brightest part of the line, and the error will be represented by the lines of lesser inclination; while if h is nearly one-fourth of the grating space, the minimum would be selected, and for values between, the measurements would be uncertain, with a tendency to setting on the "center of gravity."

This surmise is amply verified by observation. For this

purpose a grating was mounted on a spectrometer, and a pair of plane-parallel glasses was placed in front of the collimator lens. One of the plates could be rotated through a measured angle, thus introducing in the corresponding half of the field any required retardation. The results are shown in Fig. 4.

The curves from this and other observations are repeated in Fig. 3. If a vertical line, for instance that at $a = 0.15\pi$, be drawn through the curves, the intersections will correspond to errors of 4.0, 3.5, 2.0 and -1.5 twentieths respectively. The constant error introduced in comparing the second with the

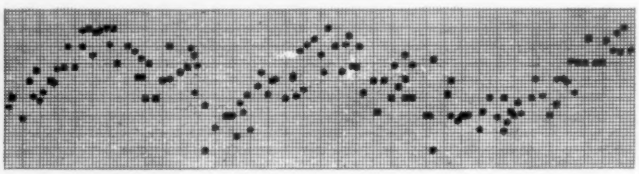


FIG. 4.

third spectrum will be 0.1, while between the third and fourth the error will amount to 0.17. If $a = \frac{1}{3}\pi$, corresponding to $h = \frac{1}{6}\sigma$, the error introduced in comparing the first spectrum with the second will be 0.5. The absolute error depends on the number of lines in the grating. If this be 100,000, these errors will be 1, 1.7, and 5 parts in a million — quantities of the same order as those given by Perot and Fabry,[1] indicating a systematic error in Rowland's table of wave-lengths.

If the ruling is affected by a periodic error, the spectral lines are accompanied by ghosts. The elementary explanation has long been known, but, so far as I know, no general theory has been proposed which furnishes a relation between the character of the periodic error and the intensity of the ghosts.

The result in its widest generality would be obtained by substituting for ϕ and ψ any periodic functions; but it will serve our purpose if ϕ is even and ψ is odd.

[1] *Comptes Rendus*, **133**, 153, 1901.

Putting $\phi(s) = \Sigma a_q \cos qs$ and $\psi(s) = \Sigma b_q \sin qs$ in (7), we obtain

$$C = \int a_q \cos(q-p)\lambda d\lambda + \int b_q \sin(q-p)\lambda d\lambda \ ,$$

$$+ \int a_q \cos(q+p)\lambda d\lambda - \int b_q \sin(q+p)\lambda d\lambda \ ,$$

$$S = 0 \ .$$

If the number of periods is large, the value of C will be appreciable only in the vicinity of $p = q$ and $p = -q$, and in these directions the intensity will be

$$I_q = C_q^2 = (a_q + b_q)^2 \quad \text{and} \quad I'_q = (a_q - b_q)^2$$

or

$$\left.\begin{aligned} I_q &= \left[\int_0^{2\pi} \phi(s) \cos qs\, ds + \int_0^{2\pi} \psi(s) \sin qs\, ds\right]^2 \\ I'_q &= \left[\int_0^{2\pi} \phi(s) \cos qs\, ds - \int_0^{2\pi} \psi(s) \cos qs\, ds\right]^2 \end{aligned}\right\} \qquad (13)$$

If $b_q = 0$, the ghosts are symmetrical about the principal line. If $a_q = b_q$, the ghosts on the side of $p = -q$ vanish.

If the only error is that due to unequal spacing, let $\phi(s) = \cos \omega$ and $\psi(s) = \sin \omega$, then, if $q = \frac{2\pi m}{S}$, where S is the period of the error, and $m = 1, 2, 3 \ldots$

$$a_q = \int_0^S \cos \omega \cos qs\, ds \ , \quad b_q = \int_0^S \sin \omega \sin qs\, ds \ ,$$

$$I_q = \left[\int_0^S \cos(\omega - qs)\, ds\right]^2 , \quad I'_q = \left[\int_0^S \cos(\omega + qs)\, ds\right]^2 .$$

As an illustration take $\omega = hs$ from 0 to S; then

$$I_m = \left(\frac{\sin hS}{hS - 2\pi m}\right)^2 \quad \text{and} \quad I'_m = \left(\frac{\sin hS}{hS + 2\pi m}\right)^2 .$$

The expression (4) represents the intensity of the reflected light, whether the surface be furrowed or not. For normal incidence, therefore, the intensity of the diffraction image is [1]

$$I = \left[\int ds \cos(2my \cos\theta - ms \sin\theta)\right]^2 + \left[\int ds \sin(2my \cos\theta - ms \sin\theta)\right]^2 .$$

[1] A more general expression including both absorption and retardation is $I = C^2 + S^2$, where

$$C = \int f(s) \cos(\omega - ps) ds \quad \text{and} \quad S = \int f(s) \sin(\omega - ps) ds \ .$$

But if we replace $2my \cos \theta$ by ω and $m \sin \theta$ by p, this is the same as (9), save that p is now single-valued. Hence all the preceding results except those relating to the form of the furrow may be applied to the case of smooth surfaces such as mirrors and prisms.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO,
October 1903.

SOLAR PROMINENCES AND TERRESTRIAL MAGNETISM.

By A. L. CORTIE, S.J.

RECENT researches with regard to the relation which exists between Sun-spots and terrestrial magnetism, while confirming the general connection between the two classes of phenomena established by Mr. Ellis and Canon Spée, have emphasized the conclusion that this relation is not one of efficient cause and effect. Such is the outcome of the comparison of the magnetic storms recorded at Stonyhurst and the greater solar spots made by Father Sidgreaves, and the analysis of the Kew magnetograms for the period of eleven years recently concluded and published by Dr. Chree. The same conclusion was set forth in a former paper by the writer on "Minimum Sun-spots and Terrestrial Magnetism,"[1] which contained a detailed study of the individual Sun-spots and the magnetic curves for the period 1899–1901. There is an undoubted general connection, not a mere series of coincidences, between Sun-spots and magnetic storms, but the supposed direct action of Sun-spots is in individual cases so capricious and irregular that it is doubtful whether they can be regarded as even instrumental causes of the magnetic storms. The existence rather of a common cause is indicated by the observations, which acts sometimes on the Sun causing spots, sometimes on the magnets, and sometimes on both Sun and magnets.

But the question has been raised more than once, and recently in a paper on "The Relation between Solar Prominences and Terrestrial Magnetism" by Sir Norman and Dr. Lockyer,[2] whether solar prominences may not supply the places of spots in those cases in which great or active magnetic storms occur without the presence of any spot. The curves presented in the paper show, as was to be expected, that at times of Sun-spot maximum activity the disturbance is general, and extends even

[1] ASTROPHYSICAL JOURNAL, **16**, 203–210, 1902. [2] *Proc. R. S.*, **71**, 244, 1903.

to prominences in high solar latitudes, the curve of annual frequency for such prominences bearing a very striking similarity to that of great magnetic disturbances; that is, disturbances in which the range of the declination magnet is greater than 60′, as deduced from Mr. Ellis's observations. But we would submit that this fact does not show that solar prominences, and in particular those in high solar latitudes, possess any special virtue or efficacy in the causation of magnetic storms not possessed by Sun-spots, and does not warrant, of itself, the conclusion that prominences may cause magnetic storms in the absence of Sun-spots. Rather is it not only one manifestation of the increased solar external activity which is displayed in greater and more numerous spots, and increase in number, latitude, and extent of both prominences and coronal streamers?

Observations of the prominences and chromosphere were taken at Stonyhurst during the years 1880–92, and in the *Observatory* for March 1893 will be found the curve of profile area of the prominences, compared with the curve of Sun-spot surface deduced from the Greenwich records for the whole period of the observations. The accord of the two curves is very striking, and corroborates what was known before, that the activity of the prominences is in direct proportion to the activity of Sun-spots. Now, both Canon Spée and Mr. Ellis have shown the complete similarity of the curves for Sun-spot frequency and magnetic declination over long periods. Mr. Ellis, too, has demonstrated that the Sun-spot curve is also identical with the curve of frequency of magnetic storms. Hence it follows that the accord of the prominence curves and the magnetic curves is no proof of the causality of prominences with regard to magnetic storms. If it were possible to make a curve of isolated solar prominences and then show that such a curve, to the exclusion of the Sun-spot curve, corresponded to the curve of magnetic storm frequency, then the argument would have a greater force. But, except that they occur in latitude in which Sun-spots are not found, solar prominences are not phenomena isolated from spots and faculæ. Nay more, it is only one class of solar prominences that are even separated in latitude from Sun-spots, namely, the relatively

quiet, as distinguished from the eruptive, prominences. For, with regard to this latter class, they are almost entirely confined to the Sun-spot zones, and as a general rule accompany outbursts of spots or faculæ. To take one period as an instance, namely, that treated of by Professor Wolfer in the *Publicationen der Sternwarte des Eidg. Polytechnikums zu Zürich* (Band III). Of 315 metallic or eruptive prominences included in the lists discussed for the years 1893–95, 274, or 86 per cent., were connected with spots; 27, or 9 per cent., with faculæ; while only 14, or 5 per cent., were independent of spots and faculæ. Father Fenyi, too, in the publications from the Haynald Observatory at Kalocsa, calls attention to the same fact, though the converse is not true, as spots and faculæ have frequently no accompanying eruptive prominences.

Nevertheless, to quote from Sir Norman and Dr. Lockyer's paper: "The magnitude of magnetic storms appears to vary according to the particular position as to latitude of the prominence on the Sun's disk. The nearer the poles (either north or south) the prominence occurs, the greater the magnetic storm, and these are the regions where no spots exist." As a contribution to the elucidation of this point, it was determined to study in detail all the more noteworthy prominences of some selected period, and to see whether they exercised any effect upon the magnets independently of the spots and faculæ. Obviously a minimum period of solar activity is the best for this purpose, first because in a maximum period it is impossible to unravel the various outbursts of spots, faculæ, and prominences, and apportion to them their proper magnetic storms; and, secondly, because, if in a time of minimum Sun-spots an extraordinary prominence appears, and prominences are to be supposed, for the sake of argument, to be the cause of magnetic storms, it ought to be accompanied by at least an active movement of the magnets. It was intended to study the four years 1887–90, for which an excellent series of observations with detailed descriptions of the greater outbursts has been published by Father Fenyi at Kalocsa, but as the work of comparison is rather laborious, the results for only two years have been gathered together in the following table,

which contains details with regard to forty-eight prominences. The principle of selection has been to admit such as were especially noteworthy either on account of their eruptive nature, or the displacements of their spectrum lines, or were of a height equal to or greater than 100." Their character as eruptive or non-eruptive is indicated in the third column, their maximum height or displacement and their heliographic co-ordinates being given in other columns of the table. In the columns devoted to the readings of the curves of the declination magnet, the first gives the maximum diurnal range for the date on which the prominence was observed, the second the intensity of the storm, if any, observed within three days before or after the prominence was seen, and the third the date on which the magnetic storm occurred. In Mr. Ellis's discussion of Sun-spot frequency and magnetic storms an active storm on the declination magnet is one in which the range exceeds 30′ and is less than 60′, while a great storm is one in which the range exceeds 60′. We have used the corresponding numbers, 3, 4, etc., as indications of intensity. In the last column are given the Greenwich numbers for such spot-groups as have been identified as connected with any of the prominences.

Of these forty-eight prominences twenty-nine were either immediately associated with spots and faculæ, or occurred in the latitudes frequented by Sun-spots. This number includes all the so-called metallic prominences. During the year 1887 there were only two noteworthy Sun-spot groups on the Sun; those numbered thirty-two and thirty-four in the Stonyhurst series. The first was born on the visible disk on May 14, and after five rotations disappeared, also while on the visible disk, on August 4, but in the next rotation there was an outburst in such close proximity to its position as evidently to have formed part of the same disturbance. This one disturbance covers the groups numbered 1978, 1987, 1992, 1998, 2006, and 2010 at Greenwich. Its mean longitude was 92° and its mean latitude −8°. At its first appearance it crossed the western limb on May 22, and the table shows that there was a fine metallic prominence seen on the limb, which also showed a displacement corresponding to no less

NOTEWORTHY PROMINENCES, 1887–88.

No.	Date	Character	Maximum Height	Maximum Displacement	Heliographic Longitude	Heliographic Latitude	Magnetic Declination			Greenwich Nos. Spots or Faculæ
							Diurnal Range	Storm		
								Intensity	Date	
	1887			km						
1	May 22	M.	49″	426	90°	− 8° W	10′.0	0	....	1978
2	June 6	H.	...	135	75	−12 E	7.0	0	....	1987 Faculæ
3	8	M.	24	...	48	− 1 E	3.1	0	....	1988
4	23	H.	137	...	30	+31 W	5.9	3	June 20	....
5	25	H.	20	70	180	− 7 E	7.5	0	....	1991?
6	26	M.	22	...	169	− 8 E	8.0	0	....	1991
7	27	H.	105	...	335	−11 W	8.0	0	....	....
8	July 1	H.	165	320	275	− 6 W	8.8	0	....	Faculæ?
9	9	M.	51	85	175	−13 W	15.8	0	....	1991
10	16	H.	113	...	265	−47 E	13.7	0	....	....
11	16	H.	122	...	85	+ 8 W	13.7	0	....	Faculæ?
12	18	M.	...	...	58	+ 2 W	20.7	0	....	1994
13	19	H.	125	...	45	+22 W	11.0	0	....	....
14	20	H.	119	...	210	−17 E	9.2	0	....	....
15	29	H.	246	...	270	+47 W	9.2	3	Aug. 1	....
16	30	M.	...	...	259	−12 W	9.0	3	Aug. 2	1995 Faculæ
17	30	M.	...	...	79	− 6 E	9.0	3	Aug. 2	1999
18	30	H.	92	284	280	−10 W	9.0	3	Aug. 2	1995
19	Aug. 12	M.	...	...	267	− 6 E	6.3	0	....	2001
20	25	M.	...	...	97	− 7 E	12.8	0	....	2006 Faculæ
21	29	M.	...	...	223	+ 1 W	31.3	3	Aug. 28	2005
22	29	H.	106	...	40	+21 E	31.3	3	Aug. 29	....
23	Sept. 15	M.	...	...	0	− 7 W	16.1	0	....	Faculæ
24	18	H.	...	184	320	−45 W	11.0	0	... .	....
25	Oct. 12	H.	108	...	0	+46 W	16.9	0	....	....
	1888									
26	Jan. 10	H.	...	85	76	− 8 E	3.0	3	Jan. 7	2029
27	Feb. 1	H.	...	412	326	−10 W	1.0	0	....	....
28	7	H.	126	...	68	+18 E	1.0	0	....	....
29	17	M.	...	133	295	− 2 E	....	0	....	2035
30	March 6	H.	...	113	58	+11 E	9.0	3	March 9	Faculæ
31	7	H.	...	57	224	−49 W	15.6	3	March 9	... .
32	14	M.	24	...	311	− 8 E	3.1	0	....	2039
33	20	M.	...	150	235	− 9 E	3.0	0	....	2041
34	April 6	H.	100	...	179	−17 W	13.3	3	April 4	Faculæ?
35	16	H.	101	...	238	−37 E	12.5	3	April 13	....
36	29	M.	...	...	246	− 8 W	11.0	0	....	2051
37	May 11	H.	...	102	266	− 6 E	0.0	3	May 7	2052?
38	June 7	H.	...	120	271	+58 E	11.2	4	June 3	....
39	July 13	H.	127	...	265	−22 W	9.5	0	....	2059 Faculæ
40	28	H.	...	145	314	− 7 E	15.5	0	....	2061
41	Aug. 22	H.	100	...	325	+20 E	6.5	0	....	....
42	Sept. 5	M.	151	...	147	−10 E	2.0	0	....	2070
43	6	M.	158	297	147	−10 E	2.0	0	....	2070
44	10	M.	38	...	272	− 8 W	4.0	0	....	2066
45	Oct. 13	H.	101	...	197	−47 W	14.9	0	....	....
46	25	H.	152	...	220	+34 E	1.5	0	....	....
47	Nov, 26	M.	27	...	159	− 9 E	16.1	0	....	2076
48	30	H	120	...	104	−23 E	11.8	0	....	....

a velocity than 426 km per second in the line of sight. The maximum diurnal range on the day was 10′, and there was no disturbance of the magnets that could be classed as active. The other prominences of the table connected with this spot-group are No. 2, seen on its second entrance on the disk, again without magnetic storm, and No. 20, also without storm of any kind, the maximum diurnal range being 7′.0 and 12′.8 respectively. The first magnetic storm which occurs at any date near to that on which a prominence was seen was on June 20, a fine isolated prominence being observed on the twenty-third. If this is to be considered as a case of connection of isolated prominences and magnetic storm, the next big prominence, possibly connected with a facula, but having no connection with spots, beyond its occurrence in the spot-zones, is directly opposed to any such connection. It was a splendid prominence which changed its form very rapidly. In eleven minutes it rose from 60″ to 163″, with a corresponding line-of-sight displacement of the C line indicating a velocity of 320 km per second. In about seventeen minutes after attaining its maximum height it had practically disappeared. More than this, as the table shows, there were many fine prominences observed by Father Fenyi in July, and Canon Spée's tables deduced from observations at Rome and Palermo also show an increase in number of prominences during this month. Yet the magnets were very quiet, and remarkably so as the end of the month approached. However, at the end of the month, on the twenty-ninth and thirtieth, four fine prominences were observed, three connected with Sun-spot groups, one of which, that numbered eighteen, was active all day, and one an isolated hydrogen prominence in higher latitudes. The magnets were actively disturbed on August 1 and 2. Is the connection here, if any, with the spots or with the one isolated prominence? Again we have storms on the magnets on August 28 and 29, and on the twenty-ninth two fine prominences are observed, one connected with a Sun-spot, the other isolated. Which is to have the credit of the magnetic storm? On January 7 there was a magnetic storm, and on January 10 a fine prominence connected with group 2029 of the Greenwich

series, while the magnetic storm of March 9 may be credited to the isolated prominence of March 7, or to that connected with the faculæ of March 6. The most telling cases, however, in favor of isolated prominences affecting the magnets are those numbered 35 and 38 in the list. On April 16 the spotted area of the Sun was extremely small, and so too was the area covered by faculæ; therefore the storm of the thirteenth may possibly be connected with the prominence of the sixteenth. Similar remarks apply to the prominence of June 7, on which day there were no spots on the Sun, and a moderate area of faculæ, and the magnetic storm of June 3, allowing four days' grace, for a prominence which at the time of the storm must have been on the Sun's invisible hemisphere. But the most active prominence of the year was observed on September 5 and 6, in connection, however, with a spot-group of moderate size, No. 2070 in the Greenwich series, and which, according to the Stonyhurst records, was one of three distinct formations occurring in the same position. Its presence is also shown in the table in No. 47, connected with spot-group 2076. There is not the slightest corresponding movement of any sort on the magnets in connection with the big September prominences, the curves being straight lines, for several days before and after, and only a slight movement in connection with the November prominence. Finally on October 13 and 25, and November 30, 1888, there occur cases of isolated prominences higher than that of April 16, with which we connected the magnetic storm of April 13. The magnets, however, are quiet, the greatest range being 16.′1 on November 26. It follows, therefore, that in the period discussed, which, though limited in extent, contains many fine prominences, no conclusion can be drawn as to the relationship of prominences and magnetic storms, seeing that in the cases of both eruptive and non-eruptive prominences, though some few seem to be connected with magnetic storms, others equally high and equally active are totally unconnected with any extraordinary movements of the needles.

STONYHURST COLLEGE OBSERVATORY,
September 1903.

THE SPECTRUM OF LIGHTNING.

By PHILIP FOX.

SPECTRA of lightning flashes were photographed on the nights of July 16, 17, August 3, and October 6 by means of an objective-prism spectroscope. The camera lens was of 35 mm aperture and 274 mm focal length. The 30° flint-glass prism was from a large spectroscope loaned to the Observatory by the Massachusetts Institute of Technology. Of the dozen plates showing spectra the best was obtained on August 3 at $9^h\ 10^m$ in the evening, and shows three flashes. They are reproduced herewith (Plate IX). The original negative is on a Cramer Isochromatic plate.

Vogel and Lohse,[1] and Schuster[2] identified certain lines in the lightning spectrum with lines of the spark spectrum of air. Following the suggestion of their work, the spectroscope was provided with a slit and collimator, and the air spectrum, obtained by passing a spark between silver terminals, was photographed. The agreement in position of the lines is shown in Fig. 1. The wave-lengths of the lines, with their identification, are given in the table, which also shows the wave-lengths of lines as determined by Vogel and Lohse, Schuster, and E. C. Pickering.[3] Lieutenant Herschel[4] also identified some of the lightning lines, the principal one probably being that at λ 5003 of the present determination.

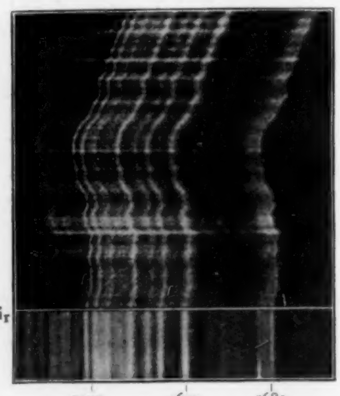


FIG. 1.—Spectrum of lightning and of air.

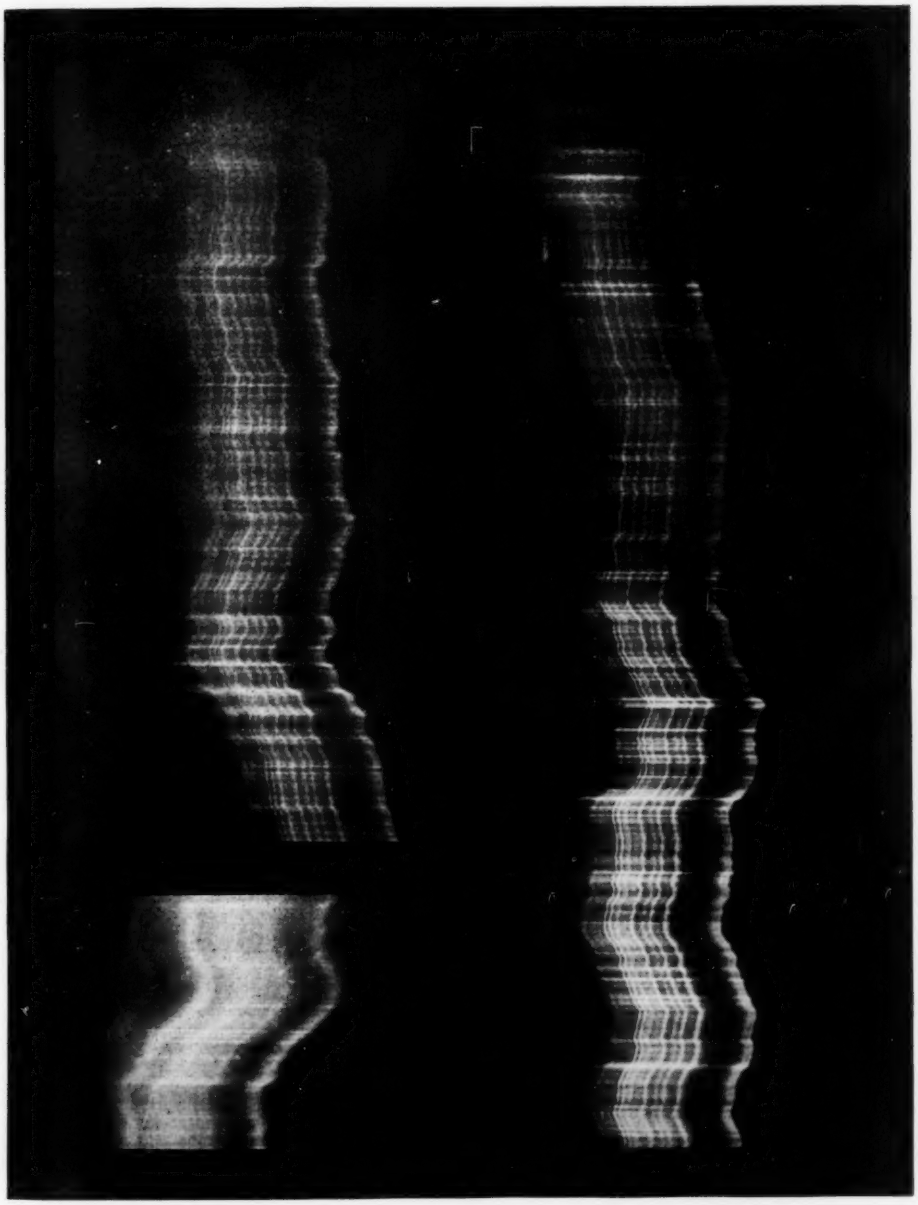
[1] VOGEL, *Pogg. Ann.*, **143**, 653–654, 1871.

[2] *Phil. Mag.*, (5) **7**, 316–321, 1879.

[3] *Harvard College Observatory Circular* No. 62.

[4] *Proc. R. S.*, **17**, 61, 1869.

PLATE IX.



2

1

3

SPECTRUM OF LIGHTNING FLASHES.

TABLE.

Measured Wave-Length		Character	Air Spectrum (Neovius)	Vogel	Schuster	Pickering
High Power	Low Power					
3848	3838	Faint, broad	3839.8 3843.1 3845.3 3850.6 3857.2			
3898	3890 3915	Faint, double at times	3882.6 3912.2 3919.2			3881
3950	3943	Very faint	3945.3 3947.5 3954.6 3956.1			3956
3997	3997	Sharp	3995.2			3998 } $H\epsilon$
4041.5 s	4041,5 s	Sharp	4041.5			4046
4074	4077	Poor } coalesce at times	4072.4 4076.3 4097.3			
4106	4105	Fair }	4103.4 4105.2 4119.4			4102 $H\delta$
4143 v. edge 4165 center 4183 r. edge	4154 4183	Broad, double at times	4152.0 4153.7 4176.7 4180.3 4185.8			4147 4187
4236	4238	Strong	4228.9 4237.0 4242.1			4222 4263
4349	4359	Broad, fair	4349.4			4341
4439	4439	Strong	4415.0 4417.3 4447.3			
4529	4535	Broad, hazy	4530.3			4519
4630.7 s	4603. v.edge 4630.7 center 4660. r. edge	Broad, strong	4596.6 to 4661.9 maximum 4630.9	4583 } 4673 } band		4643
4790	4786	Very faint	4788.5			4754
4858	4842	Very faint	4861.0	4860		4861 $H\beta$
5003.7 s	5003.7 s	Good	5002.7 5005.7	5002	5002	4940 5022
5175	5156	Very faint	5180	5184	5181	5173
5306 v. edge 5600 max. 5683 r. edge		Broad band, red edge very strong	includes 5453.8 5462.8 5479.8 5496.6 5535.2 5667.1 5679.8	5341	5260 5334 5592 5681	5595

The letter s indicates the lines used as standards in deriving the formula for wave-lengths. The higher power used in the microscope magnified about fifteen times, the lower about three times.

In studying the spectra some curious facts were observed. The relative intensity of some lines with regard to their fellows is found to vary in different parts of the spectrum. In the first flash, the line at λ4349 diminishes rapidly in relative intensity from the cloud to the ground. The line, a combination of λ4074 and λ4106, at the top of the flash is one of the heaviest lines, being much stronger than its two neighbors to the left, λ4041.5 and λ3997. Toward the ground, however, it has lost greatly in relative intensity, having been surpassed by both λ4041.5 and λ3997. These two have increased from very faint lines to rank among the strongest.

In comparing the first flash with the second, it is found that in the second these same changes do not exist. The only case of change in relative intensity is shown by the line at λ4439, which increases slightly toward the ground.

The third flash, which was at the very edge of the plate, shows the line at λ3848 very strong, while in the other flashes it is comparatively faint. This flash shows a line far to the violet of λ3848.

This work has been conducted under the direction of Mr. Hale. The writer is indebted to Mr. Ellerman for assistance in the laboratory.

Yerkes Observatory,
October 12, 1903.

Minor Contributions and Notes.

PHOTOGRAPHIC SPECTRUM OF *NOVA GEMINORUM*.[1]

At the suggestion of Director Campbell, I obtained the spectrum of the *Nova* with the Crossley reflector, using the small slitless spectrograph. Six negatives were obtained on the night of April 2, with exposures ranging from 10 seconds to 19 minutes. Thirty seconds showed the stronger lines very faintly, and five minutes gave a good negative. The accompanying reproductions, Figs. 2 and 3, are chiefly from the negative having an exposure of ten minutes, although all the plates were made use of in determining the relative intensities. Fig. 2 shows the spectrum as a negative.

The plates used were ordinary commercial dry plates, sensitive only to blue and violet radiations. The recorded spectrum of *Nova Geminorum* consists of bright lines and bands superposed on a continuous spectrum, and extends from $H\beta$ to $\lambda 335$. The general appearance of the spectrum resembles somewhat the April 1901 spectrum of *Nova Persei* obtained by Campbell and Wright with the Mills spectrograph, in the region where the two instruments give comparable results. As no observations of the ultra-violet spectrum of *Nova Persei* were secured before September 1901, when the star had become a nebula, we do not know the early history of the lines at $\lambda 339$ and $\lambda 346$, photographed by Mr. Stebbins. These lines are certainly not yet developed in the case of *Nova Geminorum*, although there is a decided strengthening in the spectrum at about this point. There is very little similarity in the spectra of *Nova Persei* in September 1901 and *Nova Geminorum* in the region above $H\delta$. Lines occupying approximately the positions of $H\epsilon$ and $H\zeta$, as well as those at $\lambda\lambda 339$ and 346, are the strongest lines in *Nova Persei*, but are indicated in *Nova Geminorum* only by slight strengthenings in the continuous spectrum. $H\beta$ and $H\delta$ are strong in the recent *Nova*, but very weak in the later spectrum of *Nova Persei*. The chief nebular line at $\lambda 501$ is not shown in the spectrum of *Nova Geminorum*, but was conspicuous in *Nova Persei*.

It is altogether probable that these differences between the spectrum of *Nova Geminorum* and the later spectrum of *Nova Persei* are

[1] From *Lick Observatory Bulletin* No. 37.

due to the different stages of development of the two stars. As *Nova Geminorum* assumes the nebular state, we may expect its ultra-violet spectrum to conform more and more to that shown by *Nova Persei*.

Following are the positions of the lines and maxima in the spectrum determined from three of the plates:

No. 8 5 m.	1903, April 2 No. 9 10 m.	No. 10 19 m.		Description
λ	λ	λ		
486	486	486	$H\beta$	Strong. Narrow
462	462	463		Very strong. Broad
445	446	446		Faint
434	434	434	$H\gamma$	Very strong
410	410	410	$H\delta$	" "
397	397	397	$H\epsilon$	Moderately strong
389	389	390	$H\zeta$	Faint
	384			Max. of band $H\eta$
374	374	372		" " $H\kappa$
350	350	352		" "
335	335	333		End of spectrum

Spectrograms were secured also on April 3, 5, 6, and 8. A comparison of those taken on the 2d and 8th show changes in the character of the spectrum, in the interval of six days. The most noticeable change is in the ultra-violet, where the continuous spectrum has become weaker and the bands at λλ 350, 374, and 384, more conspicuous in consequence. There are indications also of the development of the lines at λλ 339 and 346. Below $H\delta$ there seems to be little or no change in the continuous spectrum. $H\beta$ has become weaker, however, and there are traces of radiations in the region of the chief nebular line.

The spectrum was examined visually on all the nights when spectrograms were secured. The $H\alpha$ line was always very conspicuous. A brightening was observed in the yellow at about the position of the sodium lines.

In this connection it should be said that the dispersion with this spectrograph is so small, the linear distance between $H\beta$ and λ 335 being only 3½ mm, that close lines cannot be separated with it.

Between April 1 and 8 the *Nova* decreased slightly in brightness. On the latter date it was recorded as fully as bright as the 8.6 magnitude star preceding, with which it was compared each night.

C. D. PERRINE.

APRIL 15, 1903.

PLATE X.

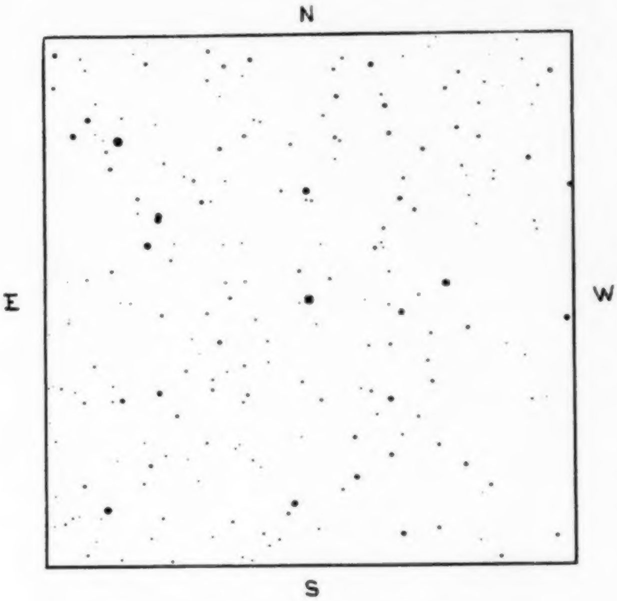


FIG. 1.—Region about *Nova Geminorum.*

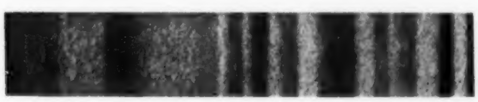


FIG. 2.—The Photographic Spectrum of *Nova Geminorum* (Negative). April 2, 1903.

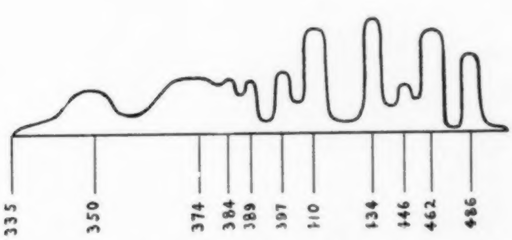


FIG. 3.—Intensity Curve.

THE SPECTRUM OF *NOVA GEMINORUM*.[1]

THE first opportunity to observe the spectrum of *Nova Geminorum* was on the evening of April 1, when its magnitude was estimated at 8.5. Only visual observations were attempted, by means of Spectrograph I, consisting of a single light flint prism in connection with the collimator and camera sections of the Mills Spectrograph.[2] These visual observations and later photographs showed that the spectrum was intermediate between that of *Nova Persei* in April 1901 and in the following July. It consists mainly of isolated bright bands, many of which coincide approximately with the lines of hydrogen. The brightest band was coincident with $H\beta$. To the less refrangible side of it there was another strong band, and a stretch of faint continuous spectrum extending just beyond the position of the D lines. There were several faint maxima near the red end of this continuous spectrum. On another night we found that one of these coincided approximately with the D lines; and the bright $H\alpha$, overlooked on the first evening, was easily visible.

To the violet of the $H\beta$ band there appeared another band, fainter and broader, and farther up there was a faint band about the position of $H\gamma$. Nothing could be seen above this.

Following is the list of good photographs secured to date:

Number	Date, G. M. T.	Exposure	Kind of Plate
2717*A*........	April 2, $17^h\ 18^m$	$4^h\ 0^m$	Cramer Crown
2818*D*........	5, 17 26	3 46	Cramer Iso. Inst.
2722*B*........	6, 17 21	3 55	Cramer Iso. Inst.
2731*A*........	8, 17 33	4 40	Cramer Iso. Inst.
2737*D*........	22, 17 12	3 0	Cramer Crown
2757*A*........	May 4, 17 9	2 36	Cramer Crown

The first photograph was secured on the evening of April 2. The auxiliary correcting lens brought all light in the $H\gamma$ region in focus on the slit, and the adjustments of the instrument were such that the camera was in fair focus from $\lambda 5000$ to the ultra-violet. The plate substantiated our results of the previous night in the region common to both visual and photographic observation. Fig. 4 is a reproduction of this plate, and Fig. 5 is its intensity curve. Table I gives a description of the details of the bands. The wave-lengths are calculated by means of the Hartmann-Cornu formula, the constants of which are determined

[1] From *Lick Observatory Bulletin* No. 37. [2] See *L. O. Bulletin*, 1, No. 8, p. 46.

from certain lines in the comparison spectrum of iron. The wave-lengths are corrected for the orbital and the diurnal motions of the Earth. The effect of curvature of the lines is negligible. They are not corrected for the velocity in the line of sight of the star itself, for we have no means of determining that velocity with certainty. The spectrum seems at first sight to resemble that of *Nova Persei* in July 1901,[1] but there are great differences in the details. The several bands in the present case have not the uniformity of structure that characterized those of *Nova Persei*, nor are the maxima and minima by any means so sharp and definite. Indeed, it is quite possible that a number of the details shown in Fig. 5 and Table I are due to accidental arrangements of the silver grains.

More important still is the apparently total absence of certain bands which were present in the spectrum of *Nova Persei*. One of these is that corresponding to the nebular line at λ3869, which in *Nova Persei* was considerably stronger than the $H\delta$ band. No trace of it appears on the present plate.

The band farthest to the violet has its maximum at λ3987. Here again is a decided difference: in *Nova Persei* this band was very strong, and was displaced to the violet of the corresponding hydrogen line $H\epsilon$ more than the other bands corresponding with hydrogen lines; and it was shown[2] that the preponderating radiation was not $H\epsilon$, but a companion nebular line, previously unobserved. In *Nova Geminorum* this band is quite faint, and the maximum is 17 tenth-meters to the red of $H\epsilon$.

In *Nova Persei* the $H\gamma$ band was superimposed on another corresponding to the nebular line λ4363. In *Nova Geminorum*, though the corresponding band shows greater evidences of complexity than any other in the spectrum, λ4363 would fall almost wholly outside of it, so that no radiations corresponding to this line exist with an intensity at all comparable with those due to $H\gamma$.

There is in *Nova Geminorum* a band with maximum at λ4625, which may perhaps be said to correspond with the λ4643 band in *Nova Persei*, but there is none corresponding to the complex band λ4669–λ4746, which was apparently formed of two or more superposed bands. In *Nova Geminorum* the nebular line λ4686 would fall just on the edge of the band that is present, and the helium line λ4713 would lie entirely outside of it. No trace of the two nebular lines λ4959 and λ5007 can be seen on this plate. No doubt this is partially due to the lower sen-

[1] See *L. O. Bulletin*, 1, No. 8. [2] *L. O. Bulletin*, 1, No. 8, p. 54.

sitiveness of the plate at this region, and to the fact that much of the light is lost at the slit, for a later negative showed a very faint band at λ 5007 when the slit was properly focused for this light and an isochromatic plate used. It may be concluded, however, that with this exception the star showed no certain traces of the so-called "nebular" lines in the early part of April.

The positions of the bands are interesting. The strong band between $H\beta$ and $H\alpha$ has its maximum 18 tenth-meters to the violet of what we may call the corresponding band in the July spectrum of *Nova Persei*. The band corresponding to $H\epsilon$ has its maximum on the other hand 17 tenth-meters to the red of the position of the hydrogen line. Hartmann[1] states that on March 31 the middle of the $H\beta$ and $H\gamma$ bands were both displaced 8 Ångström units toward the red from the positions of the corresponding hydrogen lines. Our plate shows no such displacement; $H\beta$ appears only 1½ and $H\gamma$ about ½ of a tenth-meter too far to the red, and these amounts are hardly greater than the probable error. The discordance between Hartmann's results and ours may be explained, of course, by a real change in the spectrum between March 31 and April 2; but more probably it is due to the fact that Hartmann's plate was underexposed. The limits he assigns to the bands fall well within ours in each case, and in each of the bands the more intense part is toward the red of the middle. These two facts would certainly tend to make the middle of the band appear more to the red on his plate than on ours.

Messrs. Frost and Adams, discussing a plate taken March 28[2] describe $H\gamma$ as a very faint band merging into a brighter band which extends from λ 4347 to λ 4371. As indicated in Figs. 4 and 5, our plate shows some evidence of a composite character in what we have called the $H\gamma$ band, with much greater intensity near the red border than near the violet; but, as noted above, the middle of the whole band coincides very closely with the normal position for $H\gamma$, so that we have thought it best to speak of it as a single band showing considerable detail in structure. In any case, however, the limit λ 4371 falls outside the limits of the band on our plate, in spite of the fact that this part of the spectrum is doubtless much stronger on our plate than on theirs. There seems to be some evidence, therefore, of a perceptible change in the $H\gamma$ region between March 28 and April 2.

[1] *Astronomische Nachrichten*, **161**, 324, 1903.

[2] ASTROPHYSICAL JOURNAL, **17**, 304, 1903.

A second photograph of the same range of spectrum was taken April 22. It is reproduced in Fig. 6, with the corresponding intensity curve shown in Fig. 7. Table II gives a description of the details. As in the case of the earlier negative, many of the details plotted in

TABLE I.

April 2 2717 *A*	Description	
λ 3974	Beginning	Faint band
3987	Maximum	Faint band
3992	End	Faint band
3992–4064	Faint continuous spectrum between these points	
4064	Slight increase in brightness	*Hδ* band
4078	Beginning of stronger part of band	*Hδ* band
4081	Maximum	*Hδ* band
4083	Minimum	*Hδ* band
4094	Suspect a minimum	*Hδ* band
4098	Maximum	*Hδ* band
4100	Suspect a minimum	*Hδ* band
4102	Maximum	*Hδ* band
4103.0	Minimum rather sharp	*Hδ* band
4107	Maximum	*Hδ* band
4110.9	Minimum	*Hδ* band
4116	Maximum	*Hδ* band
4116.8	Minimum	*Hδ* band
4120	Maximum	*Hδ* band
4124	End of stronger part	*Hδ* band
4144	End of weaker part	*Hδ* band
4317	Beginning of bright band	*Hγ* band
4327	Maximum	*Hγ* band
4331.7	Minimum	*Hγ* band
4334	Maximum	*Hγ* band
4336.8	Minimum	*Hγ* band
4340	Maximum, not well defined	*Hγ* band
4342.4	Minimum	*Hγ* band
4347	Maximum	*Hγ* band
4350.2	Minimum	*Hγ* band
4358	Maximum	*Hγ* band
4365	End of band	*Hγ* band
4365–4449	Continuous spectrum between these points	
4449	Beginning	Faint band
4464	Maximum	Faint band
4492	End	Faint band
4570	Faint beginning of band	Strong band
4625	Principal maximum	Strong band
4646.2	Suspect narrow bright line or maximum	Strong band
4651.4	Suspect narrow bright line or maximum	Strong band
4658.9	Suspect narrow bright line or maximum	Strong band
4695	End of band	Strong band
4839	Beginning	*Hβ* band
4866	Maximum	*Hβ* band
4887	End	*Hβ* band

PLATE XI.

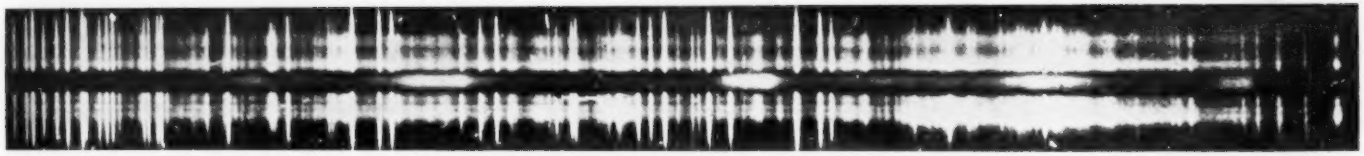


FIG. 4.—*Nova Geminorum*, enlarged ninefold. 1903, April 2.

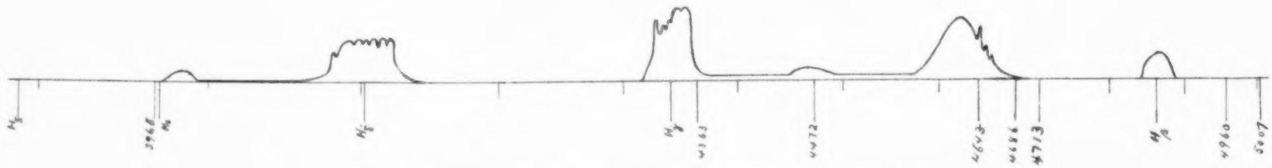


FIG. 5.—*Nova Geminorum*, Intensity Curve. 1903, April 2.

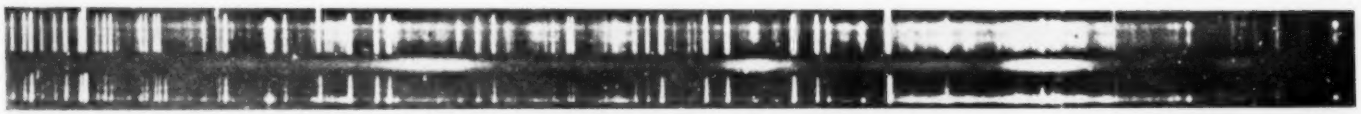


FIG. 6.—*Nova Geminorum*, enlarged ninefold. 1903, April 22.

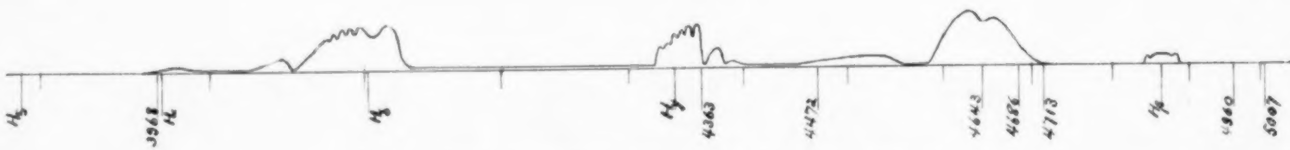


FIG. 7.—*Nova Geminorum*, Intensity Curve. 1903, April 22.

Fig. 7 and tabulated in Table II may be false effects due to chance grouping of the silver grains, though in each case the plate was measured under low magnification to avoid as far as possible errors of this kind.

TABLE II.

April 22 λ 2727 *D*	Description	
3959	Beginning of very faint continuous spectrum	
3982	Very faint maximum	
4021	Beginning of faint part of band	*Hδ* band
4046	Maximum	
4052	Minimum	
4078	Suspect a minimum	
4080	Maximum	
4082.1	Minimum	
4084	Maximum	
4086.9	Minimum	
4090	Maximum	
4093.0	Minimum	
4095	Maximum	
4107	Very broad minimum	
4116	Broad maximum	
4126	End of brighter part	
4145	End of fainter part	
4322	Beginning of bright band	*Hγ* band
4326	Maximum	
4329.4	Minimum	
4334	Maximum	
4337.3	Minimum	
4341	Maximum	
4342.4	Minimum	
4346	Maximum	
4348.9	Minimum	
4352	Maximum	
4355.2	Minimum	
4358	Maximum	
4364	End of main part of band	
4378	Faint broad maximum	
4382	End of stronger part of auxiliary band	
4392	Suspect very faint maximum	
4454	Beginning	Very faint band in continuous spectrum, extending from λ 4392 to λ 4584
4536	Maximum	
4557	End	
4584	Beginning	Broad strong band
4630	Broad maximum	
4642	Broad minimum	
4656	Broad maximum	
4714	End	
4838	Beginning	*Hβ* band
4843	Suspect a minimum	
4877	Suspect a minimum	
4884	End	

Even a casual examination showed that pronounced changes had taken place in some of the bands. In regard to the $H\beta$ band little can be said, for it is too weak on both plates to be sure of much structural detail. The middle of the strong band between $H\beta$ and $H\gamma$ had been moved about 17 tenth-meters to the red, making it more nearly coincide with the band observed at about this place in *Nova Persei* in the summer of 1901. It was certainly changed in detail also, for it showed two pronounced broad maxima on the later plate, as against one such maximum and two or three suspected narrow maxima on the earlier date. The faint band to the violet of this was also

TABLE III

April 5 2718 *D*	April 6 2722 *B*	April 8 2731 *A*	Description	
4838	4836	....	Beginning of band	
4843	....	....	Dark line?	
4852	....	....	Slight maximum	
4859.7	4859.9	....	Minimum (or dark line?)	
4866	4865	....	Slight maximum	$H\gamma$ band
4870	....	....	Slight minimum or dark line	
4874	....	....	Slight minimum or dark line	
4877	4878	....	Maximum of band. Exact position uncertain	
4886	4886	....	End of band	
....	4987	....	First trace of very faint band	
....	5015	....	Minimum?	λ 5007
....	5022	....	Beginning of second portion	
....	5030	....	End of band	
....	....	5409	First certain trace of continuous spectrum	
....	....	5480	Very slight maximum	
....	....	5493	Well defined minimum	
....	....	5516	Slight maximum suspected	
....	....	5527	Minimum?	
....	....	5557	Maximum of band	
....	....	5579	Slight minimum	
....	....	5589	Slight maximum	
....	....	5634	Faintest portion between bands	
....	....	5653	Dark line suspected. May be chance arrangement of silver grains	
....	....	5671	Maximum of band	
....	....	5678	Minimum	
....	....	5687	Maximum	
....	....	5700	End of brighter portion of band	
....	....	5708	End of band	
....	....	5727	Beginning of band	
....	....	5739	Maximum of band	
....	....	5746	Slight maximum?	
....	....	5758	Slight maximum?	
....	....	5766	Slight maximum?	
... .	....	5767	Rather well marked minimum	
....	....	5777	End of band	

changed, becoming much broader and displaced much more toward the red. This band is so faint and broad that determinations of its limits and of the position of the maximum are uncertain. The faint band near $H\epsilon$ appeared relatively still fainter than on the earlier plate, with the maximum perhaps more toward the violet; that is, more nearly coincident with $H\epsilon$. The $H\delta$ band showed some changes in structure, as indicated in the intensity-diagrams, and also a faint companion band to the violet.

The most striking change, however, was in the $H\gamma$ band, which showed a fairly strong companion band to the red. We at once suspected that this was the first appearance of the nebular line λ 4363, but the measurements locate it too far to the red of the proper position of that line. Still it is not impossible that the whole complex band is due to radiations physically connected with the nebular line and the hydrogen $H\gamma$. No further discussion of this plate seems necessary, except to say that the $H\gamma$ band seems to have grown fainter with respect to the $H\delta$ and the λ 4643 bands.

A third plate of the same region was obtained on May 4. Unfortunately, the star was by this time so far west at dark that the exposure was necessarily rather short, so that the negative is weak in spite of a somewhat wider slit than was used in the other cases. So far as it goes, however, it seems to show that no perceptible change had occurred since April 22.

Table III gives the results for the plates taken in the $H\beta$ and the D regions. These plates are Nos. 2718*D*, 2722*B*, and 2731*A*. The comparison lines used are iron and helium. In the plates for April 6 and April 8 there is evidence of slight flexure due to the necessity for carrying the exposures to so large an hour angle. On all these plates the bands show little certain evidences of structure, and some of the details plotted in the intensity curves may well be due to a fortuitous grouping of silver grains. Plates 2718*D* and 2722*D* show also traces of the band at λ 4643, but as this band is well seen on other plates, no attempt has been made to study its details on these plates. No trace is seen on Plate 2731*A* of any band at D_3 or beyond.

A comparison of these plates with those of similar regions of *Nova Persei* taken in July and August 1901, is rendered somewhat difficult, as far as the more minute details are concerned, by the comparatively diffuse and uncertain character of the structural details seen in the *Nova Geminorum* plates. In the case of $H\beta$ the maximum intensity is very clearly at the red edge of the band (λ 4878), while in *Nova Persei*

it lies at the violet edge (λ 4852). A very slight maximum is, however, to be seen at about λ 4852 in the plate of April 5. In the region from λ 5400–λ 5800 the similarity in general detail is more marked, but the band at λ 5671 is relatively much brighter in *Nova Geminorum* than in *Nova Persei* on August 11, 1901. The most striking differences in this portion of the spectrum are the faintness of the band at λ 5007 and the apparent lack of any band at D_3. λ 5007 was relatively faint in *Nova Persei* in February and March 1901, but in July and August was very much brighter than any of the bands between it and D_3. In *Nova Geminorum* it appears as a mere trace on one plate only, and in neither plate is there a trace of the band at λ 4959. Too much reliance cannot be placed on the apparent absence of a band at D_3, on account both of the faintness of the star and of the fact that the curve of sensitiveness of the plates used changes very rapidly at this point.

Figs. 8 and 9 give a view of the region covered in Plate 2731*A* and the corresponding intensity curve. Fig. 10 shows the $H\beta$ band and the extremely faint trace of λ 5007. Figs. 11 and 12 give the intensity curves for Plates 2718*D* and 2722*B*.

It will be seen from Table III that in the plates for April 5 and April 6 there is a suspected dark line at approximately λ 4860. The mean corrected shift of this line, if real, is 1.7 t. m., corresponding to a velocity of -105 km per second. The observed appearance is too faint, however, to warrant placing much reliance upon the velocity thus obtained.

The spectrum was tested on April 4 for evidences of Zeeman effects, by rotating a Nicol prism in the eyepiece of the spectroscope. The forms of the broad bright bands appeared to remain unchanged in all positions of the prism. The same tests applied to *Nova Persei* in April 1901, gave similar results.

H. M. Reese.
H. D. Curtis.

May 9, 1903.

A LIST OF FIVE STARS WHOSE VELOCITIES IN THE LINE OF SIGHT ARE VARIABLE.[1]

The following five stars with variable radial velocities, discovered with the Mills spectrograph, are additional to the forty-two already

[1] *Lick Observatory Bulletin* No. 46.

PLATE XII.

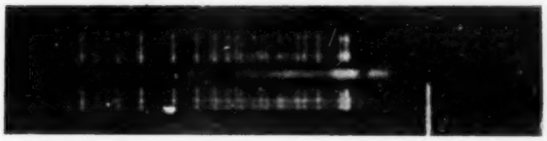


FIG. 8.—*Nova Geminorum*, Region near *D*, enlarged ninefold. 1903, April 8.

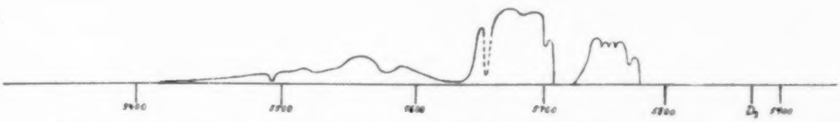


FIG. 9.—*Nova Geminorum*, Region near *D*, Intensity Curve. 1903, April 8.

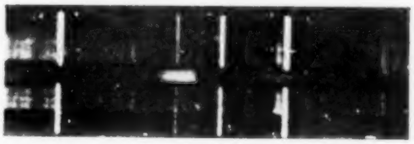


FIG. 10.—*Nova Geminorum*, $H\beta$ and λ 5007. 1903, April 8.

FIG. 11.

FIG. 12.

FIGS. 11 and 12.—*Nova Geminorum*, Intensity Curves for $H\beta$ Region. 1903, April 5 and 6.

announced. A number of the photographs and one of the measures on which these results depend are by Dr. Reese.

γ *Corvi* ($\alpha = 12^h\ 10^m.7$; $\delta = -16°\ 59'$).

Date	Velocity	Measured by
1902, December 30	+ 2 km	Curtis
1903, February 23	0	Curtis
May 6	−20	Curtis
May 11	− 6	Curtis
May 17	+ 4	Curtis

The variation was discovered from the third plate. The line at $H\gamma$ is very fair with proper exposure; those due to helium and magnesium are good.

η *Virginis* ($\alpha = 12^h\ 14^m.8$; $\delta = -0°\ 6'$).

Date	Velocity	Measured by
1903, May 17	+ 17 km	Curtis
May 24	+ 4	Curtis

$H\gamma$ is fair and $\lambda 4481$ good. There are in addition scattered iron lines, narrow and quite good.

α *Draconis* ($\alpha = 14^h\ 1^m.7$; $\delta = +64°\ 51'$).

Date	Velocity	Measured by
1902, June 16	± 0 km	Stebbins
	+ 2	Curtis
1903, April 29	−43	Curtis
May 24	−42	Curtis

The variable velocity was discovered from the second plate. The measures depend on two lines only, a rather broad $H\gamma$ and a strong magnesium.

ϵ *Herculis* ($\alpha = 16^h\ 56^m.5$; $\delta = +30°\ 4'$).

Date	Velocity	Measured by
1903, May 24	−70 km	Curtis
May 27	−34	Curtis

In this star $H\gamma$ is very broad; the magnesium line a $\lambda 4481$ is also rather broad. The measures depend on the magnesium line alone.

δ *Aquilae* ($\alpha = 19^h\ 20^m.5$; $\delta = +2°\ 55'$).

Date	Velocity	Measured by
1900, May 22	−25 km	Curtis
1902, July 31	−35	Reese
1903, May 12	− 2	Curtis
May 27	−32	Curtis

The iron lines are diffuse and broad; both $H\gamma$ and $\lambda 4481$ are broad and hard to measure. The first plate is a very poor one and the given velocity can be considered only as an approximation.

W. W. CAMPBELL.
HEBER D. CURTIS.

JUNE 11, 1903.